EE-6201 CIRCUIT THEORY

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This is to certify that the e-course material

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being prepared by me and it meets the knowledge requirement of the university curriculum.

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SEAL
OBJECTIVES:

- To introduce electric circuits and its analysis
- To impart knowledge on solving circuits using network theorems
- To introduce the phenomenon of resonance in coupled circuits.
- To educate on obtaining the transient response of circuits.
- To Phasor diagrams and analysis of three phase circuits

UNIT I BASIC CIRCUITS ANALYSIS

UNIT II NETWORK REDUCTION AND NETWORK THEOREMS FOR DC AND AC CIRCUITS
Network reduction: voltage and current division, source transformation – star delta conversion.
Thevenins and Novton & Theorem – Superposition Theorem – Maximum power transfer theorem –Reciprocity Theorem.

UNIT III RESONANCE AND COUPLED CIRCUITS

UNIT IV TRANSIENT RESPONSE FOR DC CIRCUITS
Transient response of RL, RC and RLC Circuits using Laplace transform for DC input and A.C. with sinusoidal input – Characterization of two port networks in terms of Z, Y and h parameters.

UNIT V THREE PHASE CIRCUITS
Three phase balanced / unbalanced voltage sources – analysis of three phase 3-wire and 4-wire circuits with star and delta connected loads, balanced & un balanced – phasor diagram of voltages and currents – power and power factor measurements in three phase circuits.

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UNIT I
BASIC CIRCUITS ANALYSIS

1.1.INTRODUCTION:
The interconnection of various electric elements in a prescribed manner comprises as an electric circuit in order to perform a desired function. The electric elements include controlled and uncontrolled source of energy, resistors, capacitors, inductors, etc. Analysis of electric circuits refers to computations required to determinethe unknown quantities such as voltage, current and power associated with one or more elements in the circuit. To contribute to the solution of engineering problems one must acquire the basic knowledge of electric circuit analysis and laws. Many other systems, like mechanical, hydraulic, thermal, magnetic and power system are easy to analyze and model by a circuit. To learn how to analyze the models of these systems, first one needs to learn the techniques of circuit analysis. We shall discuss briefly some of the basic circuit elements and the laws that will help us to develop the background of subject.

1.2. BASIC ELEMENTS & INTRODUCTORY CONCEPTS:
Electrical Network:
A combination of various electric elements (Resistor, Inductor, Capacitor, Voltage source, Current source) connected in any manner what so ever is called an electrical network. We may classify circuit elements in two categories, passiveand active elements.

Passive Element:
The element which receives energy (or absorbs energy) and then either converts it into heat (R) or stored it in an electric (C) or magnetic (L) field is called passive element.

Active Element:
The elements that supply energy to the circuit is called active element. Examples of active elements include voltage and current sources, generators, and electronic devices that require power supplies. A transistor is an active circuit element, meaning that it can amplify power of a signal. On the other hand, transformer is not an active element because it does not amplify the power level and power remains same both in primary and secondary sides. Transformer is an example of passive element.

Bilateral Element:
Conduction of current in both directions in an element (example: Resistance; Inductance; Capacitance) with same magnitude is termed as bilateral element.

Unilateral Element:
Conduction of current in one direction is termed as unilateral (example: Diode, Transistor) element.

Meaning of Response:
An application of input signal to the system will produce an output signal, the behavior of output signal with time is known as the response of the system.

Potential Energy Difference:
The voltage or potential energy difference between two points in an electric circuit is the amount of energy required to move a unit charge between the two points.

Ohm’s Law: Ohm’s law states that the current through a conductor between two points is directly proportional to the potential difference or voltage across the two points, and inversely proportional to the resistance between them. The mathematical equation that describes this relationship is:

$$ I = \frac{V}{R} $$

where I is the current through the resistance in units of amperes, V is the potential difference measured across the resistance in units of volts, and R is the resistance of the conductor in units of ohms. More specifically, Ohm’s law states that the R in this relation is constant, independent of the current.

1.3. KIRCHOFF’S LAW
Kirchoff’s First Law - The Current Law, (KCL)
"The total current or charge entering a junction or node is exactly equal to the charge leaving the node as it has no other place to go except to leave, as no charge is lost within the node”.

In other words the algebraic sum of ALL the currents entering and leaving a node must be equal to zero,

$$ I(\text{exiting}) + I(\text{entering}) = 0. $$

This idea by Kirchoff is known as the Conservation of Charge.
Here, the 3 currents entering the node, I1, I2, I3 are all positive in value and the 2 currents leaving the node, I4 and I5 are negative in value. Then this means we can also rewrite the equation as;

\[ I_1 + I_2 + I_3 - I_4 - I_5 = 0 \]

**Kirchoff's Second Law - The Voltage Law, (KVL)**

"In any closed loop network, the total voltage around the loop is equal to the sum of all the voltage drops within the same loop" which is also equal to zero. In other words the algebraic sum of all voltages within the loop must be equal to zero. This idea by Kirchoff is known as the Conservation of Energy.

Starting at any point in the loop continue in the same direction noting the direction of all the voltage drops, either positive or negative, and returning back to the same starting point. It is important to maintain the same direction either clockwise or anti-clockwise or the final voltage sum will not be equal to zero.

We can use Kirchoff's voltage law when analyzing series circuits.
1.4. PROBLEMS AND CALCULATIONS:

Problem 1:
A current of 0.5 A is flowing through the resistance of 10Ω. Find the potential difference between its ends.

Solution:
Current $I = 0.5\, \text{A}$.
Resistance $R = 10\, \Omega$
Potential difference $V = ?$

$V = IR$
$= 0.5 \times 10$
$= 5\, \text{V}$.

Problem 2
A supply voltage of 220V is applied to a 100 Ω resistor. Find the current flowing through it.

Solution:
Voltage $V = 220\, \text{V}$ Resistance $R = 100\, \Omega$
Current $I = \frac{V}{R}$
$= \frac{220}{100}$
$= 2.2\, \text{A}$.

Problem 3
Calculate the resistance of the conductor if a current of 2A flows through it when the potential difference across its ends is 6V.

Solution:
Current $I = 2\, \text{A}$.
Potential difference $V = 6$.
Resistance $R = \frac{V}{I}$

$= \frac{6}{2}$
$= 3\, \text{ohm}$.

Problem 4
Calculate the current and resistance of a 100 W, 200V electric bulb.

Solution:
Power, $P = 100\, \text{W}$
Voltage, \( V = 200 \text{V} \)  
Power \( p = VI \)

\begin{align*}
\text{Current } I &= \frac{P}{V} \\
&= \frac{100}{200} \\
&= 0.5 \text{A} \\
\text{Resistance } R &= \frac{V}{I} \\
&= \frac{200}{0.5} \\
&= 400 \text{W}.
\end{align*}

**Problem: 5**

Calculate the power rating of the heater coil when used on 220V supply *taking 5 Amps*.

**Solution:**

Voltage, \( V = 220 \text{V} \) 
Current, \( I = 5 \text{A} \), 
Power, \( P = VI \)

\begin{align*}
&= 220 \times 5 \\
&= 1100 \text{W} \\
&= 1.1 \text{ KW}.
\end{align*}

**Problem: 6**

A circuit is made of 0.4 \( \Omega \) wire, a 150\( \Omega \) bulb and a 120\( \Omega \) rheostat connected in series. Determine the total resistance of the resistance of the circuit.

**Solution:**

Resistance of the wire = 0.4\( \Omega \)  
Resistance of bulb = 150\( \Omega \)  
Resistance of rheostat = 120\( \Omega \)

In series,

Total resistance, \( R = 0.4 + 150 + 120 \)

\[ R = 270.4 \Omega \]

**Problem: 7**

Three resistances of values 2\( \Omega \), 3\( \Omega \) and 5\( \Omega \) are connected in series across 20 V.D.C supply. Calculate (a) equivalent resistance of the circuit (b) the total current of the circuit (c) the voltage drop across each resistor and (d) the power dissipated in each resistor.

**Solution:**

Total resistance \( R = R_1 + R_2 + R_3 \)

\[ \begin{align*}
&= 2 + 3 + 5 = 10 \Omega \\
\text{Voltage} &= 20 \text{V} \\
\text{Total current } I &= \frac{V}{R} = \frac{20}{10} = 2 \text{A}. \\
\text{Voltage drop across 2} \Omega \text{ resistor } V_1 &= IR_1 \\
&= 2 \times 2 = 4 \text{ volts}. \\
\text{Voltage drop across 3} \Omega \text{ resistor } V_2 &= IR_2 \\
&= 2 \times 3 = 6 \text{ volts}. \\
\text{Voltage drop across 5} \Omega \text{ resistor } V_3 &= IR_3 \\
&= 2 \times 5 = 10 \text{ volts}. \\
\text{Power dissipated in 2} \Omega \text{ resistor is } P_1 &= I_2 R_1 \\
&= 22 \times 2 = 8 \text{ watts}. \\
\text{Power dissipated in 3 resistor is } P_2 &= I_2 R_2 \\
&= 22 \times 3 = 12 \text{ watts}. \\
\text{Power dissipated in 5 resistor is } P_3 &= I_2 R_3 \\
&= I_2 R_3
\end{align*} \]
= 22 × 5 = 20 watts.

**Problem: 8**
A lamp can work on a 50 volt mains taking 2 amps. What value of the resistance must be connected in series with it so that it can be operated from 200 volt mains giving the same power.

**Solution:**
Lamp voltage, \( V = 50 \text{V} \)
Current, \( I = 2 \text{ amps} \)

Resistance of the lamp = \( V/I = 50/25 = 25 \Omega \)

Resistance connected in series with lamp = \( r \)

Supply voltage = 200 volt.

Circuit current \( I = 2 \text{A} \)

Total resistance \( R_t = V/I = 200/2 = 100 \Omega \)

\[ R_t = R + r = 100 = 25 + r \]

\[ r = 75 \Omega \]

**Problem: 9**
Find the current flowing in the 40Ω Resistor, R3

**Solution:**
The circuit has 3 branches, 2 nodes (A and B) and 2 independent loops.
Using Kirchoff’s Current Law, KCL the equations are given as;
At node A: \( I_1 + I_2 = I_3 \)
At node B: \( I_3 = I_1 + I_2 \)

Using Kirchoff’s Voltage Law, KVL the equations are given as;
Loop 1 is given as: \( 10 = R_1 \times I_1 + R_3 \times I_3 = 10I_1 + 40I_3 \)
Loop 2 is given as: \( 20 = R_2 \times I_2 + R_3 \times I_3 = 20I_2 + 40I_3 \)
Loop 3 is given as: \( 10 - 20 = 10I_1 - 20I_2 \)

As \( I_3 \) is the sum of \( I_1 + I_2 \) we can rewrite the equations as;
Eq. No 1: \( 10 = 10I_1 + 40(I_1 + I_2) = 50I_1 + 40I_2 \)
Eq.No 2: \( 20 = 20I_1 + 40(I_1 + I_2) = 40I_1 + 60I_2 \)
We now have two "Simultaneous Equations" that can be reduced to give us the value of both \( I_1 \) and \( I_2 \). Substitution of \( I_1 \) in terms of \( I_2 \) gives us the value of \( I_1 \) as -0.143 Amps.

Substitution of \( I_2 \) in terms of \( I_1 \) gives us the value of \( I_2 \) as +0.429 Amps.

As: \( I_3 = I_1 + I_2 \)

The current flowing in resistor \( R_3 \) is given as: \(-0.143 + 0.429 = 0.286 \) Amps

and the voltage across the resistor \( R_3 \) is given as : \( 0.286 \times 40 = 11.44 \) volts.

**Problem: 10**

Find the current in a circuit using Kirchhoff’s voltage law.

![Circuit Diagram]

**Solution:**

\[
80 = 20(I) + 10(I)
\]

\[
80 = 30(I)
\]

\[
I = \frac{80}{30} = 2.66 \text{ amperes}
\]

**1.5. DC CIRCUITS:**

- A DC circuit (Direct Current circuit) is an electrical circuit that consists of any combination of constant voltage sources, constant current sources, and resistors. In this case, the circuit voltages and currents are constant, i.e., independent of time. More technically, a DC circuit has no memory. That is, a particular circuit voltage or current does not depend on the past value of any circuit voltage or current. This implies that the system of equations that represent a DC circuit do not involve integrals or derivatives.

- If a capacitor and/or inductor is added to a DC circuit, the resulting circuit is not, strictly speaking, a DC circuit. However, most such circuits have a DC solution. This solution gives the circuit voltages and currents when the circuit is in DC steady state. More technically, such a circuit is represented by a system of differential equations. The solutions to these equations usually contain a time varying or transient part as well as constant or steady state part. It is this steady state part that is the DC solution. There are some circuits that do not have a DC solution. Two simple examples are a constant current source connected to a capacitor and a constant voltage source connected to an inductor.

- In electronics, it is common to refer to a circuit that is powered by a DC voltage source such as a battery or the output of a DC power supply as a DC circuit even though what is meant is that the circuit is DC powered.
1.6. AC CIRCUITS:
Fundamentals of AC:
- An alternating current (AC) is an electrical current, where the magnitude of the current varies in a cyclical form, as opposed to direct current, where the polarity of the current stays constant.
- The usual waveform of an AC circuit is generally that of a sine wave, as these results in the most efficient transmission of energy. However in certain applications different waveforms are used, such as triangular or square waves.
- Used generically, AC refers to the form in which electricity is delivered to businesses and residences. However, audio and radio signals carried on electrical wire are also examples of alternating current. In these applications, an important goal is often the recovery of information encoded (or modulated) onto the AC signal.

1.7. DIFFERENCE BETWEEN AC AND DC:
Current that flows continuously in one direction is called direct current. Alternating current (A.C) is the current that flows in one direction for a brief time then reverses and flows in opposite direction for a similar time. The source for alternating current is called a.c generator or alternator.

Cycle:
- One complete set of positive and negative values of an alternating quantity is called cycle.

Frequency:
- The number of cycles made by an alternating quantity per second is called frequency. The unit of frequency is Hertz (Hz)

Amplitude or Peak value:
- The maximum positive or negative value of an alternating quantity is called amplitude or peak value.

Average value:
- This is the average of instantaneous values of an alternating quantity over one complete cycle of the wave.

Time period:
- The time taken to complete one complete cycle.

Average value derivation:
Let \( i = \) the instantaneous value of current and \( i = Im \sin \phi \)

Where, \( Im \) is the maximum value.

Resistors in series and parallel circuits:

Series circuits:
Figure shows three resistors \( R_1 \), \( R_2 \) and \( R_3 \) connected end to end, i.e. in series, with a battery source of \( V \) volts. Since the circuit is closed a current \( I \) will flow and the p.d. across each resistor may be determined from the voltmeter readings \( V_1 \), \( V_2 \) and \( V_3 \)

In a series circuit
(a) the current I is the same in all parts of the circuit and hence the same reading is found on each of
the two ammeters shown, and
(b) the sum of the voltages V1, V2 and V3 is equal to the total applied voltage, V, i.e.
\[ V = V_1 + V_2 + V_3 \]
From Ohm’s law:
\[ V_1 = IR_1, \quad V_2 = IR_2, \quad V_3 = IR_3 \text{ and } V = IR \]
where R is the total circuit resistance.
Since \( V = V_1 + V_2 + V_3 \)
then \( IR = IR_1 + IR_2 + IR_3 \)
Dividing throughout by I gives
\[ R = R_1 + R_2 + R_3 \]
Thus for a series circuit, the total resistance is obtained by adding together the values of the separate
resistances.

**Problem 1:** For the circuit shown in Figure 5.2, determine (a) the battery voltage \( V \), (b) the total
resistance of the circuit, and (c) the values of resistance of resistors \( R_1 \), \( R_2 \) and \( R_3 \), given that the
p.d.’s across \( R_1 \), \( R_2 \) and \( R_3 \) are 5V, 2V and 6V respectively.

Diagram 1

(a) Battery voltage \( V = V_1 + V_2 + V_3 \)
\[ = 5 + 2 + 6 = 13 \text{V} \]
(b) Total circuit resistance \( R = \frac{V}{I} \)
\[ = \frac{13}{4} = 3.25 \Omega \]
(c) Resistance \( R_1 = \frac{V_1}{I} \)
\[ = \frac{5}{4} \]
\[ = 1.25 \text{Ω} \]
Resistance \( R_2 = \frac{V_2}{I} \)
\[ = \frac{2}{4} \]
\[ = 0.5 \text{Ω} \]
Resistance \( R_3 = \frac{V_3}{I} \)
\[ = \frac{6}{4} \]
\[ = 1.5 \text{Ω} \]

**Problem 2.** For the circuit shown in Figure determine the p.d. across resistor \( R_3 \). If the total
resistance of the circuit is 100\( \_\), determine the current flowing through resistor \( R_1 \). Find also the
value of resistor \( R_2 \).

Diagram 2
P.d. across R3, $V_3 = 25 - 10 - 4 = 11V$

Current $I = \frac{V}{R}$

$= \frac{25}{100}$

$= 0.25A$, which is the current flowing in each resistor

Resistance $R_2 = \frac{V_2}{I}$

$= \frac{4}{0.25}$

$= 16 \Omega$

**Problem 3:** A 12V battery is connected in a circuit having three series-connected resistors having resistances of 4 $\Omega$, 9 $\Omega$ and 11 $\Omega$. Determine the current flowing through, and the p.d. across the 9 $\Omega$ resistor. Find also the power dissipated in the 11 $\Omega$ resistor.

Total resistance $R = 4 + 9 + 11 = 24 \Omega$

Current $I = \frac{V}{R}$

$= \frac{12}{24}$

$= 0.5A$, which is the current in the 9 $\Omega$ resistor.

P.d. across the 9 $\Omega$ resistor, $V_1 = I \times 9 = 0.5 \times 9$

$= 4.5V$

Power dissipated in the 11 $\Omega$ resistor, $P = I^2R = 0.5^2 \times 11$

$= 0.25 \times 11$

$= 2.75W$

1.8. PARALLEL NETWORKS:

**Problem 1:** Figure shows three resistors, $R_1$, $R_2$ and $R_3$ connected across each other, i.e. in parallel, across a battery source of $V$ volts.

In a parallel circuit:
(a) the sum of the currents $I_1$, $I_2$ and $I_3$ is equal to the total circuit current, $I$, i.e. $I = I_1 + I_2 + I_3$, and
(b) the source p.d., $V$ volts, is the same across each of the
resistors.
From Ohm’s law:
\[ I_1 = \frac{V}{R_1} \]
\[ I_2 = \frac{V}{R_2} \]
\[ I_3 = \frac{V}{R_3} \]
and \[ I = \frac{V}{R} \]
where \( R \) is the total circuit resistance.
Since \( I = I_1 + I_2 + I_3 \)
then
\[ \frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \]
Dividing throughout by \( V \) gives:

\[
\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}
\]

This equation must be used when finding the total resistance \( R \) of a parallel circuit. For the special case of two resistors in parallel

\[
R = \frac{R_1 R_2}{R_1 + R_2}
\]

**Problem 2:** For the circuit shown in Figure , determine (a) the reading on the ammeter, and (b) the value of resistor \( R_2 \).

P.d. across \( R_1 \) is the same as the supply voltage \( V \).
Hence supply voltage, \( V = 8 \times 5 = 40 \text{V} \)
(a) Reading on ammeter, \( I = \frac{V}{R} = 40/20 = 2 \text{A} \)
(b) Current flowing through \( R_2 = 11 - 8 - 2 = 1 \text{A} \)
Hence, \( R_2 = \frac{V}{I_2} = 40/1 = 40 \Omega \)
(a) The total circuit resistance $R$ is given by
\[
\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{3} + \frac{1}{6}
\]
Hence, $R = 6/3 = 2\ \Omega$
(b) Current in the 3 $\Omega$ resistance, $I_1 = V$
$R_1 = 12/3 = 4A$

**Problem 3:** For the circuit shown in Figure find (a) the value of the supply voltage $V$ and (b) the value of current $I$.

(a) P.d. across 20 $\Omega$ resistor = $I_2R_2 = 3 \times 20 = 60V$, hence supply voltage $V = 60V$ since the circuit is connected in parallel.
(b) Current $I_1 = V/R_1 = 60/10 = 6A$; $I_2 = 3A$
$I_3 = V/R_3 = 60/60 = 1A$
Current $I = I_1 + I_2 + I_3$ and hence $I = 6 + 3 + 1 = 10A$
Alternatively,
\[
\frac{1}{R} = \frac{1}{60} + \frac{1}{20} + \frac{1}{10} = 1 + 3 + 6/60 = 10/60
\]
Hence total resistance $R = 6010 = 6 \Omega$
Current $I = V/R = 60/6 = 10A$

**Problem 4:** Find the equivalent resistance for the circuit shown in Figure

$R_3$, $R_4$ and $R_5$ are connected in parallel and their equivalent resistance $R$ is given by:
\[
\frac{1}{R} = \frac{1}{1/3} + \frac{1}{1/6} + \frac{1}{1/18} = 6 + 3 + 1/18 = 10/18
\]
Hence $R = 18/10 = 1.8 \Omega$

The circuit is now equivalent to four resistors in series and the equivalent circuit resistance $= 1 + 2.2 + 1.8 + 4 = 9 \Omega$

### 1.9. MESH ANALYSIS:
This is an alternative structured approach to solving the circuit and is based on calculating mesh currents. A similar approach to the node situation is used. A set of equations (based on KVL for each mesh) is formed and the equations are solved for unknown values. As many equations are needed as unknown mesh currents exist.

Step 1: Identify the mesh currents
Step 2: Determine which mesh currents are known
Step 2: Write equation for each mesh using KVL and that includes the mesh currents
Step 3: Solve the equations

**Step 1:**
The mesh currents are as shown in the diagram on the next page

**Step 2:**
Neither of the mesh currents is known

![Circuit Diagram](image)

**Step 3:**
KVL can be applied to the left hand side loop. This states the voltages around the loop sum to zero. When writing down the voltages across each resistor Ohm’s law is used. The currents used in the equations are the mesh currents.

\[ I_1R_1 + (I_1 - I_2)R_4 - V = 0 \]

KVL can be applied to the right hand side loop. This states the voltages around the loop sum to zero. When writing down the voltages across each resistor Ohm’s law is used. The currents used in the equations are the mesh currents.

\[ I_2R_2 + I_2R_3 + (I_2 - I_1)R_4 = 0 \]

**Step 4:**
Solving the equations we get

\[
I_1 = \frac{V}{\frac{R_2 + R_3 + R_4}{R_1R_2 + R_1R_3 + R_1R_4 + R_2R_4 + R_3R_4}}
\]

\[
I_2 = \frac{V}{R_2 + R_3 + R_4 + R_2R_4 + R_3R_4}
\]

The individual branch currents can be obtained from these mesh currents and the node voltages can also be calculated using this information. For example:
Problem 1:

Use mesh-current analysis to determine the current flowing in (a) the 5Ω resistance, and (b) the 1Ω resistance of the d.c. circuit shown in Figure.

Using Kirchhoff’s voltage law:

For loop 1, \((3 + 5) I_1 - I_2 = 4 \) .................................\((1)\)

For loop 2, \((4 + 1 + 6 + 5) I_2 - (5) I_1 - (1) I_3 = 0\) ........................................\((2)\)

For loop 3, \((1 + 8) I_3 - (1) I_2 = -5 \) .................................\((3)\)

Thus

\(8I_1 - 5I_2 - 4 = 0\)

\(-5I_1 + 16I_2 - I_3 = 0\)

\(-I_2 + 9I_3 + 5 = 0\)
Using determinants,

\[
\begin{vmatrix} I_1 & -I_2 \\ -5 & 0 & -4 \\ 16 & -1 & 0 \\ -1 & 9 & 5 \end{vmatrix} = \begin{vmatrix} -I_2 & I_3 \\ 8 & 0 & -4 \\ -5 & -1 & 0 \\ 0 & 9 & 5 \end{vmatrix} = \begin{vmatrix} I_3 \\ -1 \\ 8 & -5 & 0 \\ -5 & 16 & -1 \\ 0 & -1 & 5 \end{vmatrix}
\]

\[
\begin{vmatrix} I_1 & -10 & -4 \\ -5 & 16 & -1 \\ 9 & 5 & -1 \end{vmatrix} = \begin{vmatrix} -I_2 & 1 \\ 8 & 16 & -1 \\ 9 & 5 & 0 \\ 0 & 1 \end{vmatrix}
\]

\[
\begin{vmatrix} I_1 \end{vmatrix} = \frac{-5(-5) - 4(143)}{-547} = \frac{-I_2}{8(-5) - 4(-45)} = \frac{I_3}{-4(5) + 5(103)} = \frac{-1}{8(143) + 5(-45)}
\]

\[
I_1 = \frac{547}{919}, \quad I_2 = \frac{140}{919}, \quad I_3 = \frac{495}{919}
\]

Hence \( I_1 = \frac{547}{919} = 0.595 \) A,

\( I_2 = \frac{140}{919} = 0.152 \) A, and

\( I_3 = \frac{495}{919} = -0.539 \) A

(a) Current in the 5 \( \Omega \) resistance \( = I_1 - I_2 \)
\( = 0.595 - 0.152 \)
\( = 0.44 \) A

(b) Current in the 1 \( \Omega \) resistance \( = I_2 - I_3 \)
\( = 0.152 - (-0.539) \)
\( = 0.69 \) A
**Problem 2:** For the a.c. network shown in Figure determine, using mesh-current analysis, (a) the mesh currents $I_1$ and $I_2$ (b) the current flowing in the capacitor, and (c) the active power delivered by the $100\angle0^\circ$-V voltage source.

(a) For the first loop

$$(5-j4)I_1 - (-j4I_2) = 100\angle0^\circ \quad (1)$$

For the second loop

$$(4+j3-j4)I_2 - (-j4I_1) = 0 \quad (2)$$

Rewriting equations (1) and (2) gives:

$$
(5-j4)I_1 + j4I_2 - 100 = 0 \\
j4I_1 + (4-j)I_2 + 0 = 0
$$

Thus, using determinants,

$$I_1 = rac{-I_2}{\begin{vmatrix} j4 & -100 \\ 4-j & 0 \end{vmatrix}} = rac{-I_2}{(4j - 0) - (j4 - 0)} = \frac{-I_2}{5j} = \frac{1}{(5-j4)}I_2 \\
\Rightarrow I_1 = \frac{1}{32-j21}I_2 = 4.44 + j12 \quad \text{i.e. the current}
$$

Hence

$$I_1 = \frac{(400-j100)}{(32-j21)} = \frac{412.31\angle-14.04^\circ}{38.28\angle-33.27^\circ} = 10.77\angle19.23^\circ \, \text{A} = 10.8\angle19.2^\circ \, \text{A},$$

(Check: power i correct to one decimal place

$$I_2 = \frac{400\angle-90^\circ}{38.28\angle-33.27^\circ} = 10.45\angle-56.73^\circ \, \text{A}$$

and

$$= 10.5\angle-56.7^\circ \, \text{A},$$

Thus total power dissipated $= 579.97 + 436.81 = 1016.8\, \text{W} = 1020\, \text{W}$

**Problem 3:** Calculate current through 6Ω resistance using loop analysis. (AU-JUNE-12)
Solution:

Case(1): Consider loop ABGH; Apply KVL.
\[ 10 = 2I_1 + 4(I_1 - I_2) \]
\[ 10 = 6I_1 - 4I_2 \]  
\( \quad \text{(1)} \)

Consider loop BCFG
\[ I_2 + 6(I_2 + I_3) + 4(I_2 - I_1) = 0 \]
\[ I_1 I_2 + 6I_3 - 4I_1 = 0 \]  
\( \quad \text{(2)} \)

Consider loop CDEF
\[ 20 = 4I_3 + 6(I_2 - I_3) \]
\[ 20 = 10I_3 + 6I_2 \]  
\( \quad \text{(3)} \)

\[ D = \begin{vmatrix} 6 & -4 & 0 \\ -4 & 11 & 6 \\ 0 & 6 & 10 \end{vmatrix} \]

\[ = \begin{vmatrix} 10 \\ 0 \\ 20 \end{vmatrix} \]

\[ D = [6(110-36)+4(-40)] = 284. \]

\[ D_1 = \begin{vmatrix} 10 & -4 & 0 \\ 0 & 11 & 6 \\ 20 & 6 & 10 \end{vmatrix} \]

\[ D_1 = 10[110-36+(-120)] \]

\[ = 260 \]

\[ D_2 = \begin{vmatrix} 6 & 10 & 0 \\ -4 & 0 & 6 \\ 0 & 20 & 10 \end{vmatrix} \]
\[ D_2 = 6(-120) - 10(-40) = -320 \]

\[
D_3 = \begin{vmatrix}
6 & -4 & 10 \\
-4 & 11 & 0 \\
0 & 6 & 20 \\
\end{vmatrix}
\]

\[ D_3 = 6(220) + 4(-80) + 10(-24) \]

\[ D_3 = 760 \]

\[ I_1 = D_1/D = 260/284 = 0.915 \text{A} \]

\[ I_2 = D_2/D = -320/284 = -1.1267 \text{A} \]

\[ I_3 = D_3/D = 760/284 = 2.676 \text{A} \]

Current through 6Ω resistance = \[ I_2 + I_3 = -1.1267 + 2.676 = 1.55 \text{A} \]

**Problem 4:** Find the current through branch a-b using mesh analysis. (JUN-09)

\[ I_1 - I_2 = 5 \text{A} \]

**Solution:**

Consider loops

Loop HADE \[ 5I_1 + 2I_2 + 6(I_2 - I_3) = 60 \]

\[ 5I_1 + 8I_2 - 6I_3 = 60 \] \[ (1) \]

Loop ABCDA \[ 3I_3 + 6(I_3 - I_2) = -50 \]

\[ 3I_3 + 6I_3 - 6I_2 = -50 \]

\[ 9I_3 - 6I_2 = -50 \] \[ (2) \]

\[ I_2 - I_1 = 5 \text{A} \] \[ (3) \]

From (1), (2) & (3).

\[ D = \begin{vmatrix}
-1 & 1 & 0 \\
5 & 8 & -6 \\
0 & -6 & 9 \\
\end{vmatrix} \]
\[ D = -1(72-36)-1(45) \]
\[ D = -81. \]
\[ D_3 = \begin{vmatrix} -1 & 1 & 5 \\ 5 & 8 & 60 \\ 0 & -6 & -50 \end{vmatrix} \]
\[ = -1(-400+360)+250 +5(-30) \]
\[ = 40+250-150 \]
\[ D_3 = 140. \]

The current through branch ab is 1.7283A which is flowing from b to a.

1.10. NODAL ANALYSIS:

Nodal analysis involves looking at a circuit and determining all the node voltages in the circuit. The voltage at any given node of a circuit is the voltage drop between that node and a reference node (usually ground). Once the node voltages are known any of the currents flowing in the circuit can be determined. The node method offers an organized way of achieving this.

Approach:

Firstly all the nodes in the circuited are counted and identified. Secondly nodes at which the voltage is already known are listed. A set of equations based on the node voltages are formed and these equations are solved for unknown quantities. The set of equations are formed using KCL at each node. The set of simultaneous equations that is produced is then solved. Branch currents can then be found once the node voltages are known. This can be reduced to a series of steps:

Step 1: Identify the nodes
Step 2: Choose a reference node
Step 3: Identify which node voltages are known if any
Step 4: Identify the BRANCH currents
Step 5: Use KCL to write an equation for each unknown node voltage
Step 6: Solve the equations

This is best illustrated with an example. Find all currents and voltages in the following circuit using the node method. (In this particular case it can be solved in other ways as well)

Step 1:
There are four nodes in the circuit. A, B, C and D

Step 2:
Ground, node D is the reference node.

Step 3:
Node voltage B and C are unknown. Voltage at A is V and at D is 0

Step 4:
The currents are as shown. There are 3 different currents
Step 5:
I need to create two equations so I apply KCL at node B and node C.
The statement of KCL for node B is as follows:

\[
\frac{V - V_B}{R_1} + \frac{V_C - V_B}{R_2} + \frac{-V_B}{R_4} = 0
\]

The statement of KCL for node C is as follows:

\[
\frac{V_C - V_B}{R_2} + \frac{-V_B}{R_3} = 0
\]

Step 6:
We now have two equations to solve for the two unknowns \(V_B\) and \(V_C\). Solving the above two equations we get:

\[
V_C = V \frac{R_3 R_4}{R_1 R_2 + R_1 R_3 + R_1 R_4 + R_2 R_4 + R_3 R_4}
\]

\[
V_B = V \frac{R_4 (R_2 + R_3)}{R_1 R_2 + R_1 R_3 + R_1 R_4 + R_2 R_4 + R_3 R_4}
\]

Further Calculations
The node voltages are known all known. From these we can get the branch currents by a simple application of Ohm's Law:

\[
I_1 = (V - V_B) / R_1
\]

\[
I_2 = (V_B - V_C) / R_2
\]
Problem 1: Find the current through each resistor of the circuit shown in fig, using nodal analysis.

Solution:

At node 1,

\[-I_1 - I_2 - I_3 = 0\]
\[-[V_1 - 15/1] - [V_1][V_1 - V_2/0.5] = 0\]
\[-V_1 + 15 - V_1 - 2V_1 + 2V_2 = 0\]
\[4V_1 - 2V_2 = 15 \text{ } \text{(1)}\]

At node 2,

\[I_3 - I_4 - I_5 = 0\]
\[V_1 - V_2/0.5 - V_2/2 - V_2 - 20/1 = 0\]
\[2V_1 - 2V_2 - 0.5V_2 - V_2 + 20 = 0\]
\[2V_1 - 3.5V_2 = -20 \text{ } \text{(2)}\]

Multiplying (2) by 2 & subtracting from (1)
\[5V_2 = 55\]
\[V_2 = 11V\]
\[V = 9.25V\]
\[I_1 = V_1 - 5/1 = 9.25 - 15 = -5.75A = 5.75\]
\[I_2 = V_1/1 = 9.25A\]
\[I_3 = V_1 - V_2/0.5 = -3.5A = 3.5A \leftarrow\]
\[I_4 = V_2/2 = 5.5A\]
\[I_5 = V_2 - 20/1 = 11 - 20/1 = -9A = 9A.\]

Problem 2: For the bridge network shown in Figure determine the currents in each of the resistors. (DEC-07)
Let the current in the 2 resistor be $I_1$, and then by Kirchhoff’s current law, the current in the 14 resistor is $(I - I_1)$. Let the current in the 32 resistor be $I_2$ as shown in Figure. Then the current in the 11 resistor is $(I_1 - I_2)$ and that in the 3 resistor is $(I - I_1 + I_2)$. Applying Kirchhoff’s voltage law to loop 1 and moving in a clockwise direction as shown in Figure gives:

$$54 = 2I_1 + 11(I_1 - I_2)$$

i.e. $13I_1 - 11I_2 = 54$

Applying Kirchhoff’s voltage law to loop 2 and moving in an anticlockwise direction as shown in Figure gives:

$$0 = 2I_1 + 32I_2 - 14(I - I_1)$$

However $I = 8$ A

Hence $0 = 2I_1 + 32I_2 - 14(8 - I_1)$

i.e. $16I_1 + 32I_2 = 112$

Equations (1) and (2) are simultaneous equations with two unknowns, $I_1$ and $I_2$.

$16 \times (1)$ gives: $208I_1 - 176I_2 = 864$

$13 \times (2)$ gives: $208I_1 + 416I_2 = 1456$

$(4) - (3)$ gives: $592I_2 = 592$, $I_2 = 1$ A

Substituting for $I_2$ in (1) gives:

$13I_1 - I_1 = 54$

$I_1 = 65/I_3 = 5$ A

Hence,

the current flowing in the 2 resistor = $I_1 = 5$ A

the current flowing in the 14 resistor = $I - I_1 = 8 - 5 = 3$ A

the current flowing in the 32 resistor = $I_2 = 1$ A

the current flowing in the 11 resistor = $I_1 - I_2 = 5 - 1 = 4$ A and

the current flowing in the 3 resistor = $I - I_1 + I_2 = 8 - 5 + 1 = 4$ A

**Problem 3:** Determine the values of currents $I$, $I_1$ and $I_2$ shown in the network of Figure.
Total circuit impedance,
\[ Z_T = 5 + (8)(j6)/8 + j6 \]
= 5 + (j48)(8 - j6)/8² + 6²
= 5 + (j384 + 288)/100
= (7.88 + j3.84) or 8.776 25.98° A

Current \( I = V/Z_T \)
\[ = \frac{50\angle 0°}{8.77\angle 25.98°} \]
= 5.7066 -25.98° A

Current \( I_1 = I (j6/8 + j6) \)
\[ = (5.706\angle 5.98°) (6\angle 90°)/10\angle 36.87° \]
= 3.426\angle 27.15° A

Current \( I_2 = I (8/ (8 + j6) \)
\[ = (5.70\angle -25.98°) * 8\angle 0°/10\angle 36.87° \]
= 4.5666 -62.85° A

[Note: \( I = I_1 + I_2 = 3.42 \angle 27.15° + 4.56\angle -62.85° \]
= 3.043 + j1.561 + 2.081 - j4.058
= 5.124 - j2.497 A = 5.706 -25.98° A

**Problem 4:** For the a.c. network shown in Figure, determine the current flowing in each branch using Kirchhoff’s laws.
from which, \[ I_1 = \frac{20 + j55}{64 + j27} = \frac{58.52\angle 70.02^\circ}{69.46\angle 22.87^\circ} = 0.842\angle 47.15^\circ \text{ A} \]

\[ = (0.573 + j0.617) \text{ A} \]

\[ = (0.57 + j0.62) \text{ A, correct to two decimal places.} \]

From equation (1), \[ 5 = (9 + j12)(0.573 + j0.617) - (6 + j8)I_2 \]

\[ 5 = (-2.247 + j12.429) - (6 + j8)I_2 \]

from which, \[ I_2 = \frac{-2.247 + j12.429 - 5}{6 + j8} \]

\[ = \frac{14.39\angle 120.25^\circ}{10\angle 53.13^\circ} \]

\[ = 1.439\angle 67.12^\circ \text{ A} = (0.559 + j1.326) \text{ A} \]

\[ = (0.56 + j1.33)\text{ A, correct to two decimal places.} \]

The current in the \((6 + j8)\Omega\) impedance,

\[ I_1 - I_2 = (0.573 + j0.617) - (0.559 + j1.326) \]

\[ = (0.014 - j0.709)\text{ A or } 0.709\angle -88.87^\circ \text{ A} \]

An alternative method of solving equations (1) and (2) is shown below, using determinants.

\[ (9 + j12)I_1 - (6 + j8)I_2 - 5 = 0 \quad (1) \]

\[-(6 + j8)I_1 + (8 + j3)I_2 - (2 + j4) = 0 \quad (2) \]
Problem 5: For the a.c. network shown in Figure determine, using mesh-current analysis, (a) the mesh currents $I_1$ and $I_2$ (b) the current flowing in the capacitor, and (c) the active power delivered by the 100 $\Omega$ voltage source.

Thus

\[
\begin{vmatrix}
(8 + j3) & -(2 + j4) \\
-(6 + j8) & (9 + j12) \\
\end{vmatrix} = \begin{vmatrix}
-5 & -I_2 \\
-(6 + j8) & -(2 + j4) \\
\end{vmatrix} = \begin{vmatrix}
1 & 0 \\
(9 + j12) & -(6 + j8) \\
\end{vmatrix}
\]

\[
\frac{I_1}{(20 + j40) + (40 + j15)} = \frac{-I_2}{(30 - j60) - (30 + j40)} = \frac{1}{(36 + j123) - (-28 + j96)}
\]

\[
\frac{I_1}{20 + j55} = \frac{-I_2}{-j100} = \frac{1}{64 + j27}
\]

Hence \( I_1 = \frac{20 + j55}{64 + j27} = \frac{58.52^\circ}{69.46^\circ} \)

\( = 0.842^\circ\) A

\[
\text{and} \quad I_2 = \frac{100^\circ}{69.46^\circ} = 1.440^\circ A
\]

The current flowing in the \((6 + j8)\) $\Omega$ impedance is given by:

\[
I_1 - I_2 = 0.842^\circ - 1.440^\circ A
\]

\( = (0.013 - j0.709) \text{ A or } 0.709^\circ A\)
(a) For the first loop $(5 - j4)I_1 - (-j4I_2) = 100\angle0^\circ$  \hspace{1cm} (1)

For the second loop $(4 + j3 - j4)I_2 - (-j4I_1) = 0$  \hspace{1cm} (2)

Rewriting equations (1) and (2) gives:

\begin{align*}
(5 - j4)I_1 + j4I_2 - 100 &= 0 \quad (1') \\
j4I_1 + (4 - j)I_2 + 0 &= 0 \quad (2')
\end{align*}

Thus, using determinants,

\begin{align*}
\begin{vmatrix}
j4 & -100 \\
(4 - j) & 0
\end{vmatrix}
&= \begin{vmatrix}
5 - j4 & -100 \\
j4 & 0
\end{vmatrix}
\begin{vmatrix}
5 - j4 & j4 \\
j4 & 4 - j
\end{vmatrix}
\\
I_1 & = \frac{-I_2}{j400} = \frac{1}{(32 - j21)}
\end{align*}

Hence \( I_1 = \frac{(400 - j100)}{(32 - j21)} = \frac{412.31\angle-14.04^\circ}{38.28\angle-33.27^\circ} \)

\( = 10.77\angle19.23^\circ \) A = \(10.8\angle-19.2^\circ \) A, correct to one decimal place

\( I_2 = \frac{400\angle-90^\circ}{38.28\angle-33.27^\circ} = 10.45\angle-56.73^\circ \) A

\( = 10.5\angle-56.7^\circ \) A, correct to one decimal place

(b) Current flowing in capacitor = \( I_1 - I_2 \)

\( = 10.77\angle19.23^\circ - 10.45\angle-56.73^\circ \)

\( = 4.44 + j12.28 = 13.1\angle70.12^\circ \) A.

i.e., the current in the capacitor is 13.1 A
Problem 6: In the network of Figure use nodal analysis to determine (a) the voltage at nodes 1 and 2, (b) the current in the $j4\ \Omega$ inductance, (c) the current in the $5\ \Omega$ resistance, and (d) the magnitude of the active power dissipated in the $2.5\ \Omega$ resistance. (AU DEC-10)

(c) Source power $P = V I \cos \phi = (100)(10.77) \cos 19.23^\circ$

$$= 1016.9\ W = 1020\ W,$$

correct to three significant figures.

(Check: power in $5\ \Omega$ resistor $= I_1^2(5) = (10.77)^2(5) = 579.97\ W$

and power in $4\ \Omega$ resistor $= I_2^2(4) = (10.45)^2(4) = 436.81\ W$

Thus total power dissipated $= 579.97 + 436.81$

$$= 1016.8\ W = 1020\ W,$$

correct to three significant figures.)
(a) At node 1, \( \frac{V_1 - 25\angle0^\circ}{2} + \frac{V_1}{-j4} + \frac{V_1 - V_2}{5} = 0 \)

Rearranging gives:

\[
\left( \frac{1}{2} + \frac{1}{-j4} + \frac{1}{5} \right) V_1 - \left( \frac{1}{5} \right) V_2 - \frac{25\angle0^\circ}{2} = 0
\]

i.e.,

\[ (0.7 + j0.25)V_1 - 0.2V_2 - 12.5 = 0 \]  \( 1 \)  \( 1 \)

At node 2, \( \frac{V_2 - 25\angle90^\circ}{2.5} + \frac{V_2}{j4} + \frac{V_2 - V_1}{5} = 0 \)

Rearranging gives:

\[-\left( \frac{1}{5} \right) V_1 + \left( \frac{1}{2.5} + \frac{1}{j4} + \frac{1}{5} \right) V_2 - \frac{25\angle90^\circ}{2.5} = 0 \]

i.e.,

\[-0.2V_1 + (0.6 - j0.25)V_2 - j10 = 0 \]  \( 2 \)

Thus two simultaneous equations have been formed with two unknowns, \( V_1 \) and \( V_2 \). Using determinants, if

\[ (0.7 + j0.25)V_1 - 0.2V_2 - 12.5 = 0 \]  \( 1 \)

and \[-0.2V_1 + (0.6 - j0.25)V_2 - j10 = 0 \]  \( 2 \)

then

\[
\begin{vmatrix}
\quad V_1 \\
-0.2 & -12.5 \\
(0.6 - j0.25) & -j10
\end{vmatrix}
= \frac{-V_2}{\begin{vmatrix}
(0.7 + j0.25) & -12.5 \\
-0.2 & -j10
\end{vmatrix}}
\]

\[
= \frac{1}{\begin{vmatrix}
(0.7 + j0.25) & -0.2 \\
-0.2 & (0.6 - j0.25)
\end{vmatrix}}
\]
i.e.,
\[
\frac{V_1}{(j2 + 7.5 - j3.125)} = \frac{-V_2}{(-j7 + 2.5 - 2.5)} = \frac{1}{(0.42 - j0.175 + j0.15 + 0.0625 - 0.04)}
\]
and
\[
\frac{V_1}{7.584\angle-8.53^\circ} = \frac{-V_2}{-7\angle90^\circ} = \frac{1}{0.443\angle-3.23^\circ}
\]
Thus voltage, \( V_1 = \frac{7.584\angle-8.53^\circ}{0.443\angle-3.23^\circ} = 17.12\angle-5.30^\circ \text{ V} \)
\[= 17.1\angle-5.3^\circ \text{ V}, \text{ correct to one decimal place,} \]
and voltage, \( V_2 = \frac{7\angle90^\circ}{0.443\angle-3.23^\circ} = 15.80\angle93.23^\circ \text{ V} \)
\[= 15.8\angle93.2^\circ \text{ V}, \text{ correct to one decimal place.} \]

(b) The current in the \( j4 \ \Omega \) inductance is given by:
\[
\frac{V_2}{j4} = \frac{15.80\angle93.23^\circ}{4\angle90^\circ} = 3.95\angle3.23^\circ \text{ A flowing away from node 2} \]

(c) The current in the 5 \( \Omega \) resistance is given by:
\[
I_5 = \frac{V_1 - V_2}{5} = \frac{17.12\angle-5.30^\circ - 15.80\angle93.23^\circ}{5}
\]
i.e.,
\[
I_5 = \frac{(17.05 - j1.58) - (-0.89 + j15.77)}{5}
\]
\[= \frac{17.94 - j17.35}{5} = \frac{24.96\angle-44.04^\circ}{5}
\]
\[= 4.99\angle-44.04^\circ \text{ A flowing from node 1 to node 2} \]
(d) The active power dissipated in the 2.5 Ω resistor is given by

\[ P_{2.5} = (I_{2.5})^2(2.5) = \left( \frac{V_2 - 25\angle 90^\circ}{2.5} \right)^2 (2.5) \]

\[ = \frac{(0.89 + j15.77 - j25)^2}{2.5} = \frac{(9.273\angle -95.51^\circ)^2}{2.5} \]

\[ = 85.99\angle -191.02^\circ \]

by de Moivre's theorem

\[ = 34.4\angle 169^\circ \text{ W} \]
1. Define charge. (AU-APR08)

The total deficiency or addition of excess electrons in an atom is called its charge.

Constant charge is denoted by letter Q and charge varying with time is denoted by q or q(t).

Unit of charge is coulomb.

One coulomb of charge is defined as the charge possessed by the total number of electrons.

\[
\text{i.e. } \frac{1}{1.602 \times 10^{-19}} = 6.24 \times 10^{18} \text{ number of electrons}
\]

One coulomb = charge on \(6.24 \times 10^{18}\) electrons

2. Define current. (AU-APR08)

Movement of electrons is called current. It is also defined as flow of charges.

\[
\text{Current } I = \frac{dQ}{dt} = \frac{Q}{t}
\]

Movement of electrons always flow from negative to positive.

Unit of current is Ampere.

Current is defined as the rate of flow of charge in an electric circuit or in any medium in which charges are subjected to electric field.

\[
I = \frac{Q}{t} \text{ amperes.}
\]

DC current: The current does not vary with time is called direct current. It is denoted by I.

AC current: The current varies with time is called alternating current. It is denoted by i (or)i(t).

3. Define potential difference. (AU-APR08)

It is also called as voltage (or) electric potential.

It is defined as the energy required moving the unit of charge from one point to other.

It is also defined as the difference of electric potential between the two points of the conductor.

\[
\text{Electric potential} = \frac{\text{work done}}{\text{charge}} = \frac{W}{Q} = \frac{dW}{dQ}
\]

Unit of voltage is volt (or) \(\frac{J}{C}\).

It is denoted by letter V.

4. Define Power and Energy? (AU-APR08)(AU-DEC-10)

The rate of doing work is called power.

\[
P = VI
\]

\[
P = \frac{dW}{dt} \times \frac{dQ}{dt} = \frac{dW}{dt} \text{ Unit of power is Watts (or) } \frac{J}{S}
\]

It is denoted by letter P.

The rate of doing work with time is called power. It is also called as the capacity to do the work.

Unit of Energy is Watt-hour (or) Joules. It is denoted by letter E (or) W.

5. What is electric circuit or electric network?

The combination of various electrical elements such as resistors, capacitors and inductors along with various energy sources such as voltage and current sources is called electric circuit or electric network.

6. Define Independent source.
It is defined as the source voltage independent of current flowing through it and source current independent of voltage across it. It is indicated by circle with polarity of voltage and direction of current. It is also called as uncontrolled sources.

Types of independent of sources.

7. Define Dependent sources. (AU-MAY08)

It is defined as the voltage source or current source depends on voltage or current elsewhere in the given circuit. It is indicated by diamond shape. It is also called as controlled sources.

Types of Dependent of sources.
(i) Voltage controlled voltage source (VCVS).
(ii) Voltage controlled current source (VCCS).
(iii) Current controlled current source (CCCS).
(iv) Current controlled voltage source (CCVS).

8. Define ideal voltage source

The energy source which gives constant voltage across its terminals irrespective of the current flowing through its terminal is called ideal voltage source. At any time the value of voltage at load terminals remains same.

9. Define practical voltage source.

Practical voltage source gas small amount of resistance (Rse) in series with voltage source.

\[ V_L = V_S - I_L Rse \]

Due to Rse, the voltage across load terminals decreases slightly with increase in current.

10. Define time invariant voltage source. (AU-MAY08)

The source in which voltage or current is not varying with time is called time invariant sources. It is also called as DC sources. It is denoted by capital letters.
11. Define time variant voltage source. (AU-MAY08)

The source in which voltage or current is varying with time is called time variant sources. It is also called as AC sources. It is denoted by small letters.

12. Define resistance.

The property of opposition of flow of current is called resistance.
Unit of resistance: ohm
It is denoted by letter R.
Power dissipated in resistor \( P = I^2 R \)

13. Define inductance.

The property of opposition of flow of change in current is called inductance.
Unit of resistance: Henry
It is denoted by letter L.
Energy stored in the inductor \( E = \frac{1}{2} L i^2 \)
It stores the energy in the form of magnetic field.


The property of opposition of flow of change in voltage is called capacitance.
Unit of resistance: Farad
It is denoted by letter C.
Energy stored in the inductor \( E = \frac{1}{2} CV^2 \)
It stores the energy in the form of electrostatic field.

15. Define branch and node.

**Branch:** It is a portion of a circuit with two terminal connected to it. A branch may contain one or more elements.
16. Define mesh or loop.
   It is defined as a set of branches forming a closed path in a network.

17. Define active and passive elements. (DEC-10)
   **Active element:** Active elements are the elements which supply power or energy to the network.
   - Ex. Voltage source, current source
   **Passive element:** Passive elements are the elements which either store energy or dissipate energy in the form of heat.
   - Ex. Capacitor and inductor = store the energy
   - Resistor = dissipate the energy

18. Define lumped and distributed network. (AU-MAY08)
   **Lumped network:** A network consisting of physically separable elements such as resistor, capacitor, and inductor is known as lumped network.
   - Ex. RLC network
   **Distributed network:** A network consisting of elements that are not separable for analytical purpose is known as distributed network.
   - Ex. Transmission lines (R, L, and C is distributed along its length)

19. Define bilateral and unilateral network. (DEC-07)
   **Bilateral network:** The voltage-currant relationship is same for current flowing in either direction is called bilateral network.
   - Ex. R, L, and C
   **Unilateral network:** The network has different relationships between voltage and current for the two possible directions of current.
   - Ex. Diodes, vacuum tubes

20. Define linear and non-linear network. (DEC-07)
   **Linear network:** The relationship between voltage and current is linear, then the network is called linear network.
   - Ex. Resistance
   **Non-Linear network:** The networks which do not satisfy the linear voltage-current relationship is called non-linear network.
   - Ex. Diodes, Zener Diodes

21. State Ohm’s law.
   At constant temperature, the current flowing through the resistor is directly proportional to voltage across the resistor.
   \[ V \propto I, \ V = IR \]
   \[ R = \frac{V}{I} = \text{constant} \]
   Power dissipated in resistor \( P = I^2R = VI = \frac{V^2}{R} \)

22. Write the limitations of ohm’s law. (DEC-10)
   (i) It is not applicable to non-linear devices such as diodes, zener diodes, and voltage regulators.
(ii) It is not applicable for non-metallic conductors. Ex. silicon carbide
(iii) It is not applicable for arc lamps, electronic valves and electrolytes.

23. State Kirchoff’s current law. (JUN-09) (DEC-07) (JUN-10)

\[ I_1 + I_2 = I_3 + I_4 + I_5 \quad \text{or} \quad I_1 + I_2 - I_3 - I_4 - I_5 = 0 \]

It is also called as point law (or) Kirchoff’s first law.
It is defined as algebraic sum of currents meeting at any node is equal to zero.
(Or)
At any node, sum of incoming current is equal to sum of outgoing current

Sign convention:
- Positive: current flowing towards junction.
- Negative: current flowing away from junction.

24. State Kirchoff’s voltage law. (JUN-09) (DEC-07) (DEC-10) (JUN-10)

\[ E_1 - E_2 = IR_1 + IR_2 + IR_3 \]

It is also called as mesh law (or) loop law (or) Kirchoff’s second law.
It is defined as algebraic sum of voltages around any closed path is equal to zero.
(Or)
At any closed path, sum of voltage rise is equal to sum of voltage drop

Sign convention:
- Voltage rise: current flowing from –ve to +ve terminal of battery. It must be taken as positive.
- Voltage drop: current flowing from +ve to -ve terminal of battery. It must be taken as negative.

25. Write the characteristics of series connection of resistances.
(i) Same current flows through each resistance.
(ii) Supply voltage $V$ is the sum of individual voltage drops across each resistance.
\[ V = V_1 + V_2 + V_3 \]
(iii) Equivalent resistance is equal to sum of individual resistance.
\[ R_{eq} = R_1 + R_2 + R_3 \]
(iv) Equivalent resistance is the largest of all individual resistance.
\[ R_{eq} > R_1, R_2, R_3 \]
Ex. Decoration lamps.

UNIT II

NETWORK REDUCTION AND NETWORK THEOREMS FOR DC AND AC CIRCUITS

Network reduction: voltage and current division, source transformation – star delta conversion. Thevenins and Novton & Theorem – Superposition Theorem – Maximum power transfer theorem – Reciprocity Theorem.

2.1. NETWORK REDUCTION:
2.2. VOLTAGE AND CURRENT DIVISION:
2.3. POTENTIAL DIVIDER:
The voltage distribution for the circuit shown in Figure

\[ V_1 = \left( \frac{R_1}{R_1 + R_2} \right) V \]

\[ V_2 = \left( \frac{R_2}{R_1 + R_2} \right) V \]

The circuit shown in Figure (b) is often referred to as a potential divider circuit. Such a circuit can consist of a number of similar elements in series connected across a voltage source, voltages being taken from connections between the elements. Frequently the divider consists of two resistors as shown in Figure (b), where

\[ V_{\text{OUT}} = \left( \frac{R_2}{R_1 + R_2} \right) V_{\text{IN}} \]

A potential divider is the simplest way of producing a source of lower e.m.f. from a source of higher e.m.f., and is the basic operating mechanism of the potentiometer, a measuring device for accurately measuring potential differences.

**Problem 1:** Determine the value of voltage \( V \) shown in Figure

[Diagram of a circuit with 4 \( \Omega \) and 6 \( \Omega \) resistors and a 50 V voltage source]
Problem 2: Two resistors are connected in series across a 24V supply and a current of 3A flows in the circuit. If one of the resistors has resistances of 2 Ω determine (a) the value of the other resistor, and (b) the p.d. across the 2 Ω resistor. If the circuit is connected for 50 hours, how much energy is used?

(a) Total circuit resistance \( R = \frac{V}{I} \)

\[ R = \frac{24}{3} = 8 \, \Omega \]

Value of unknown resistance, \( R_x = 8 - 2 = 6 \, \Omega \)

(b) P.d. across 2 Ω resistor, \( V_1 = IR_1 = 3 \times 2 = 6V \)

Alternatively, from above,

\( V_1 = (R_1/R_1 + R_x)) \cdot V = (2/2 + 6) \cdot (24) = 6V \)

Energy used = power \times time

\[ = V \times I \times t \]

\[ = (24 \times 3W) \times (50 \, h) \]

\[ = 3600Wh = 3.6kWh \]

Current division:

For the circuit shown in Figure, the total circuit resistance, \( R_T \) is given by:

\[ R_T = \frac{R_1 \cdot R_2}{R_1 + R_2} \]
Problem 1: For the series-parallel arrangement shown in Figure, find (a) the supply current, (b) the current flowing through each resistor and (c) the p.d. across each resistor.

(a) The equivalent resistance $R_x$ of R2 and R3 in parallel is:

$$R_x = \frac{6 \times 2}{6} + 2$$

$$= 12/8$$

$$= 1.5 \, \Omega$$

The equivalent resistance $R_T$ of $R_1$, $R_x$ and $R_4$ in series is:

$$R_T = 2.5 + 1.5 + 4 = 8 \, \Omega$$

Supply current $I = \frac{V}{R_T}$

$$= \frac{200}{8}$$

$$= 25 \, \text{A}$$

(b) The current flowing through $R_1$ and $R_4$ is 25A

The current flowing through $R_2$

$$= \left( \frac{R_3}{R_2 + R_3} \right) I = \left( \frac{2}{6 + 2} \right) 25$$

$$= 6.25 \text{A}$$

The current flowing through $R_3$

$$= \left( \frac{R_2}{R_2 + R_3} \right) I = \left( \frac{6}{6 + 2} \right) 25$$
(c) The equivalent circuit of Figure is
p.d. across R1, i.e. V1 = IR1 = (25)(2.5) = 62.5V
p.d. across Rx, i.e. Vx = IRx = (25)(1.5) = 37.5V
p.d. across R4, i.e. V4 = IR4 = (25)(4) = 100V
Hence the p.d. across R2 = p.d. across R3 = 37.5V

Problem 2: For the circuit shown in Figure 5.23 calculate (a) the value of resistor Rx such that the total power dissipated in the circuit is 2.5kW, and (b) the current flowing in each of the four resistors.

(a) Power dissipated P = VI watts, hence 2500 = (250)(I)
i.e. I = 2500/250
   = 10A
From Ohm’s law, RT = V/I = 250/10
   = 25 Ω, where RT is the equivalent circuit resistance.
The equivalent resistance of R1 and R2 in parallel is
   = 15 Ω + 10
   = 150/25
   = 6 Ω
The equivalent resistance of resistors R3 and Rx in parallel is equal to 25 Ω – 6 Ω, i.e. 19 Ω.
There are three methods whereby Rx can be determined.

Problem 3: For the arrangement shown in Figure find the current Ix.

Commencing at the right-hand side of the arrangement shown in Figure, the circuit is gradually reduced in stages as shown in Figure.
From Figure (d), \( I = \frac{17}{4.25} = 4 \text{A} \)

From Figure (b), \( I_1 = \frac{9}{9} + 3(I) = \frac{12}{4} = 3 \text{A} \)

From Figure, \( I_x = \frac{2}{2} + 8(11) = \frac{2}{10}(3) = 0.6 \text{A} \)

**Source transformation:**

Source transformation is defined as to convert the sources for easy analysis of circuit. In mesh analysis, it is easier if the circuit has voltage sources. In nodal analysis, it is easier if the circuit has current sources.

### 2.4. VOLTAGE SOURCE TO CURRENT SOURCE TRANSFORMATION:

If voltage source is converted to current source, then the current source \( I = \frac{V}{R_{se}} \) with parallel resistance equal to \( R_{se} \).
2.5. CURRENT SOURCE TO VOLTAGE SOURCE TRANSFORMATION:
If current source is converted to voltage source, then the voltage source $V = IR_{sh}$ with series resistance equal to $R_{sh}$.

2.6. STAR DELTA CONVERSION:
In many circuit applications, we encounter components connected together in one of two ways to form a three-terminal network: the —Delta or $\Delta$ (also known as the —Pi, or $\pi$) configuration, and the —Star (also known as the —Y) configuration.
Problem 1: A star-connected load consists of three identical coils each of resistance 30 $\Omega$ and inductance 127.3 mH. If the line current is 5.08 A, calculate the line voltage if the supply frequency is 50 Hz.

Inductive reactance $\text{XL} = 2\pi f L$

$$= 2\pi (50) (127.3 \times 10^{-3})$$

$$= 40 \Omega$$

Impedance of each phase $Z_p = \sqrt{R^2 + X^2 L} = \sqrt{(302 + 402)} = 50 \Omega$

For a star connection $I_L = I_p Z_p$

Hence phase voltage $V_p = I_p Z_p = (5.08)(50) = 254 V$

Line voltage $V_L = \sqrt{3} V_p = \sqrt{3}(254) = 440 V$

Problem 2: A 415V, 3-phase, 4 wire, star-connected system supplies three resistive loads as shown in Figure. Determine (a) the current in each line and (b) the current in the neutral conductor.
(a) For a star-connected system \( V_L = \sqrt{3} V_p \)

Hence \( V_p = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 240 \text{ V} \)

Since current \( I = \text{Power} \div \text{Voltage} \) for a resistive load
then \( I_R = \frac{PR}{VR} = \frac{24000}{240} = 100 \text{ A} \)
\( I_Y = \frac{PY}{VY} = \frac{18000}{240} = 75 \text{ A} \)
and \( I_B = \frac{PB}{VB} = \frac{12000}{240} = 50 \text{ A} \)

(b) The three line currents are shown in the phasor diagram of Figure. Since each load is resistive the currents are in phase with the phase voltages and are hence mutually displaced by 120°. The current in the neutral conductor is given by:
\( I_N = I_R + I_Y + I_B \) phasorially.

Figure shows the three line currents added phasorially. Oa represents \( I_R \) in magnitude and direction. From the nose of Oa, ab is drawn representing \( I_Y \) in magnitude and direction. From the nose of ab, bc is drawn representing \( I_B \) in magnitude and direction. Oc represents the resultant, \( I_N \).

By measurement, \( I_N = 43 \text{ A} \)
Alternatively, by calculation, considering \( I_R \) at 90°, \( I_B \) at 210° and \( I_Y \) at 330°:
Total horizontal component \( = 100 \cos 90° + 75 \cos 330° + 50 \cos 210° = 21.65 \)
Total vertical component \( = 100 \sin 90° + 75 \sin 330° + 50 \sin 210° = 37.50 \)
Hence magnitude of \( I_N = \sqrt{(21.65^2 + 37.50^2)} \)
\( = 43.3 \text{ A} \)

**Problem 3:** Convert the given delta fig into equivalent star.
Problem 4: Convert the given star in fig into an equivalent delta.

\[ R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3} = 1.67 \times \frac{5}{2.5} + 1.67 + 5 = 10 \Omega \]
\[ R_{23} = 2.5 + 5 \times \frac{5}{1.67} = 15 \Omega \]
\[ R_{31} = 2.5 + 1.67 + 2.5 \times \frac{1.67}{5} = 5 \Omega \]
Problem 5: Obtain the delta connected equivalent for the star connected circuit.

\[ R_1 = 10 \Omega, R_2 = 20 \Omega, R_3 = 30 \Omega \]
\[ R_{12} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} = \frac{10 \times 20 + 20 \times 30 + 30 \times 10}{30} = 36.67 \Omega \]
\[ R_{23} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} = \frac{10 \times 20 + 20 \times 30 + 30 \times 10}{10} = 110 \Omega \]
\[ R_{31} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} = \frac{10 \times 20 + 20 \times 30 + 30 \times 10}{20} = 55 \Omega \]

Problem 6: Obtain the star connected equivalent for the delta connected circuit.

\[ R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}} = \frac{6 \times 5}{5 + 6 + 7} = 1.67 \Omega \]
\[ R_2 = \frac{R_{23} R_{12}}{R_{12} + R_{23} + R_{31}} = \frac{6 \times 7}{5 + 6 + 7} = 2.33 \Omega \]
\[ R_3 = \frac{R_{31} R_{23}}{R_{12} + R_{23} + R_{31}} = \frac{5 \times 7}{5 + 6 + 7} = 1.94 \Omega \]

2.7. THEVENIN'S THEOREM:

In circuit theory, Thevenin's theorem for linear electrical networks states that any combination of voltage sources, current sources, and resistors with two terminals is electrically equivalent to a single voltage source \( V \) and a single series resistor \( R \). For single frequency AC systems the theorem can also be applied to general impedances, not just resistors.

The procedure adopted when using Thévenin’s theorem is summarized below. To determine the current in any branch of an active network (i.e. one containing a source of e.m.f.):

(i) remove the resistance \( R \) from that branch,
(ii) determine the open-circuit voltage, \( E \), across the break,
(iii) remove each source of e.m.f. and replace them by their internal resistances and then determine the resistance, \( r \), ‘looking-in’ at the break,
(iv) determine the value of the current from the equivalent circuit shown in Figure 13.33, i.e. \( I = \frac{E}{R} + r \)

**Problem 1:** Use Thévenin’s theorem to find the current flowing in the 10 \( \Omega \) resistor for the circuit shown in Figure

Following the above procedure:
- The 10 \( \Omega \) resistance is removed from the circuit as shown in Figure
- There is no current flowing in the 5 \( \Omega \) resistor and current \( I_1 \) is given by:
  \[ I_1 = \frac{10}{R_1 + R_2} \]
  \[ = \frac{10}{2 + 8} \]
  \[ = 1 \text{A} \]
P.d. across $R_2 = I_1 R_2 = 1 \times 8 = 8 \text{V}$ Hence p.d. across AB, i.e. the open-circuit voltage across the break, $E = 8 \text{V}$

(iii) Removing the source of e.m.f. gives the circuit of Figure
Resistance, $r = R_3 + R_1 R_2 / R_1 + R_2$
$= 5 + (2 \times 8 / 2 + 8) = 5 + 1.6 = 6.6 \Omega$

(iv) The equivalent Thévenin’s circuit is shown in Figure
Current $I = E / (R + r) = 8 / 10 + 6.6 = 8 / 16.6 = 0.482 \text{A}$

**Problem 2:** For the network shown in Figure determine the current in the 0.8 $\Omega$ resistor using Thévenin’s theorem.
Following the procedure:
- The 0.8 $\Omega$ resistor is removed from the circuit as shown in Figure
- Current $I_1 = 12 / 1 + 5 + 4 = 12 / 10 = 1.2 \text{A}$

P.d. across 4 $\Omega$ resistor $= 4 I_1 = (4) (1.2) = 4.8 \text{V}$
Hence p.d. across AB, i.e. the open-circuit voltage across AB, $E = 4.8 \text{V}$
= 2.4 Ω

(iv) The equivalent Thévenin’s circuit is shown in Figure (e), from which, current $I = \frac{E}{r+R}$

$= \frac{4.8}{2.4+0.8}$

$= \frac{4.8}{3.2}$

$I = 1.5A$ = current in the 0.8 Ω resistor

Problem 3: Use Thévenin’s theorem to determine the current $I$ flowing in the 4 Ω resistor shown in Figure.

Find also the power dissipated in the 4 Ω resistor.

(i) The 4 Ω resistor is removed from the circuit as shown in Figure

(ii) Current $I_1 = \frac{E_1 - E_2}{r_1 + r_2}$

$= \frac{4 - 2}{2 + 1}$

$= \frac{2}{3}A$

P.d. across AB, $E = E_1 - I_1R_1 = 4 - \left(\frac{2}{3}\right)(2) = \frac{2}{3}V$

(iii) Removing the sources of e.m.f. gives the circuit shown in Figure (c), from which resistance

$r = 2 \times \frac{1}{2} + 1$

$= 2/3 \Omega$

(iv) The equivalent Thévenin’s circuit is shown in Figure (d), from which,

\[
\text{current, } I = \frac{E}{r + R} = \frac{\frac{2}{3}}{\frac{2}{3} + 4} = \frac{8/3}{14/3}
\]

$= \frac{8}{14}$

$= 0.571A$

$= \text{current in the 4 Ω resistor}$

Problem 4: Power dissipated in 4 Ω resistor, $P = I^2R = (0.571)^2(4) = 1.304W$ Use Thévenin’s theorem to determine the current flowing in the 3 Ω resistance of the network shown in Figure (a). The voltage source has negligible internal resistance.
i) The $3 \, \Omega$ resistance is removed from the circuit as shown in Figure (b).

(ii) The $1 \frac{2}{3} \, \Omega$ resistance now carries no current.

P.d. across $10 \, \Omega$ resistor $= (10/10 + 5)(24)$

$= 16V$

Hence p.d. across AB, $E = 16V$

(iii) Removing the source of e.m.f. and replacing it by its internal resistance means that the $20 \, \Omega$ resistance is short-circuited as shown in Figure (c) since its internal resistance is zero. The $20 \, \Omega$ resistance may thus be removed as shown in Figure (d).

![Circuit Diagrams](image)

From Figure (d), resistance,

$$r = \frac{2}{3} + 10 \times 5/10 + 5$$

$= 5 \, \Omega$

(iv) The equivalent Thévenin’s circuit is shown in Figure (e), from which current, $I = E/r + R = 16/3 + 5 = 16/8$
Problem 5: A Wheatstone Bridge network is shown in Figure (a). Calculate the current flowing in the 32 Ω resistor, and its direction, using Thévenin’s theorem. Assume the source of e.m.f. to have negligible resistance.
The $32\ \Omega$ resistor is removed from the circuit as shown in Figure (b).

The p.d. between A and C,
\[ V_{AC} = \frac{R_1}{R_1 + R_4} \times (E) = \frac{2}{2 + 11(54)} = 8.31V \]

The p.d. between B and C,
\[ V_{BC} = \frac{R_2}{R_2 + R_3} \times (E) = \frac{14}{14 + 3(54)} = 44.47V \]

Hence the p.d. between A and B,
\[ V_{AB} = 44.47 - 8.31 = 36.16V \]

Point C is at a potential of $+54V$. Between C and A is a voltage drop of 8.31V. Hence the voltage at point A is $54 - 8.31 = 45.69V$. Between C and B is a voltage drop of 44.47V. Hence the voltage at point B is $54 - 44.47 = 9.53V$. Since the voltage at A is greater than at B, current must flow in the direction A to B.

(iii) Replacing the source of e.m.f. with a short-circuit (i.e. zero internal resistance) gives the circuit shown in Figure (c). The circuit is redrawn and simplified as shown in Figure (d) and (e), from which the resistance between terminals A and B,
\[ r = \frac{2 \times 11/2 + 11+ 14 \times 3/14 + 3}{22/13 + 42/17} = \frac{1.692 + 2.471}{4.163} = 4.163\ \Omega \]

(iv) The equivalent Thévenin’s circuit is shown in Figure (f), from which, current \( I = \frac{E}{r + R_5} \)
\[ = \frac{36.16}{4.163 + 32} = 1A \]

2.8. NORTON’S THEOREM:

- Norton’s theorem states the following:
  Any two-terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a current and a parallel resistor.
- The steps leading to the proper values of \( I_N \) and \( R_N \).
- Preliminary steps:
  1. Remove that portion of the network across which the Norton equivalent circuit is found.
  2. Mark the terminals of the remaining two-terminal network.
  3. Finding \( R_N \):
     Calculate \( R_N \) by first setting all sources to zero and then finding the resultant resistance between the two marked terminals. Since \( R_N = R_{Th} \) the procedure and value obtained using the approach described for Thévenin’s theorem will determine the proper value of \( R_N \).
  4. Finding \( I_N \):
     Calculate \( I_N \) by first returning all the sources to their original position and then finding the short-circuit current between the marked terminals. It is the same current that would be measured by an ammeter placed between the marked terminals.
  5. Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

Problem 1: Use Norton’s theorem to determine the current flowing in the $10\Omega$ resistance for the circuit shown in Figure.

The branch containing the $10\ \Omega$ resistance is short-circuited as shown in Figure.
Figure (c) is equivalent to Figure (b). Hence
\[ ISC = \frac{10}{2} = 5A \]

If the 10V source of e.m.f. is removed from Figure (b) the resistance ‘looking-in’ at a break made between A and B is given by:
\[ r = 2 \times \frac{8}{2} + 8 = 1.6 \Omega \]

From the Norton equivalent network shown in Figure (d) the current in the 10 Ω resistance, by current division, is given by:
\[ I = \frac{(1.6/1.6 + 5 + 10)}{(5)} = 0.482A \]
as obtained previously in problem 7 using Thévenin’s theorem.

**Problem 2:** Use Norton’s theorem to determine the current \( I \) flowing in the 4 Ω resistance shown in Figure (a).
The 4 Ω branch is short-circuited as shown in Figure (b).

From Figure (b), $I_{SC} = I_1 + I_2 = 4A$

If the sources of e.m.f. are removed the resistance ‘looking-in’ at a break made between A and B is given by:

$$r = 2 \times 1/2 + 1 = 2/3 \, \Omega$$

From the Norton equivalent network shown in Figure (c) the current in the 4 Ω resistance is given by:

$$I = \left(\frac{2}{3} \times \frac{2}{3} + 4\right)(4) = 0.571A,$$

as obtained previously in problems 2, 5 and 9 using Kirchhoff’s laws and the theorems of superposition and Thévenin.

**QUESTION BANK**

**PART-A**

**TWO MARKS**

1. **State Thevenin’s theorem.**

   Any complex network consisting of linear, bilateral and lumped elements can be replaced by a simple circuit consisting of single voltage source in series with resistance. It is also called as *Helmholtz’s theorem*.

2. **State Norton’s theorem.**

   Any complex network consisting of linear, bilateral and lumped elements can be replaced by a simple circuit consisting of single current source in parallel with resistance.
3. **State Superposition theorem.** [NOV/DEC’04] [MAY/JUNE’O7][APR-08][JUN-10]
   In a linear, bilateral and lumped circuit elements that is energized by two or more sources, the response(current through or voltage across) in any resistor is equal to individual responses in it, when sources acts separately.

4. **State Maximum power transfer theorem.** [NOV/DEC’07] [NOV/DEC’06][APR08][JUN-12]
   It states that the maximum power transferred to the load occurs when the load resistance is equal to the source resistance (equivalent resistance).
   
   **Condition for maximum power transfer** \( R_L = R_{TH} \)

5. **State Reciprocity theorem.** [NOV/DEC’07]
   It states that in a linear, bilateral, single source circuit, the ratio of excitation to response is constant when the position of excitation and response are interchanged.

6. **Draw the Thevenin’s equivalent circuit.**

   ![Thevenin’s equivalent circuit](image)

7. **Draw the Norton’s equivalent circuit**

   ![Norton’s equivalent circuit](image)

8. **What are the applications of Thevenin’s theorem?**
   1. To find the particular branch current in an electrical network while the resistance of that branch is varied with all other elements of the network remaining unchanged.
   2. Used in sensitivity analysis.

9. **What are the advantages of Thevenin’s theorem?**
   1. Applicable to circuits containing any type of load – linear or non linear or time varying.
   2. Applicable to circuits with load containing sources.
   3. Applicable to circuits with load having initial conditions on passive elements.

10. **What are the limitations of Thevenin’s theorem?**
1. Load should be connected to network containing linear elements only.
2. There should be no controlled source or magnetic coupling with the elements of load.

11. What are the applications of Maximum power transfer theorem? (AU-MAY08)
1. It is used for impedance matching.
2. It is used in communication circuits, where the power or circuit currents are low.

12. What are the limitations of Reciprocity theorem?
1. Only one source is present in the network.
2. Initial conditions should be zero.
3. The network is linear.
4. Impedance matrix is symmetric.
5. Dependent sources present in the network, even if they are linear, are excluded.

13. under what conditions, the super position theorem may be applied to the circuit. (JUN-09)
It is applicable to all time variant linear networks. It holds good for all possible locations, types and waveforms of the independent systems. The theorem applies both in time domain and frequency domain.

15. Write short notes about superposition theorem.
It is valid only for linear circuits. It is not valid for power responses. When the superposition theorem is applied to any circuit, the dependent voltage source in the circuit is always active.

16. State under what circumstances Thevenin’s equivalent circuit is useful.
Thevenin’s equivalent circuit is useful in the situation in which it is desirable to find a particular branch current in a network as the resistance of that branch is varied while all other resistances and sources remain constant.

17. What is meant by short and open circuits?
A short is conductor with zero resistance and an open is an infinitely large resistance. When we short circuit the resistance we replace the resistance by a conductor with zero resistance and when we open the circuit a resistance we remove the resistance.

18. Find the current through 10Ω resistor in the following circuit.

\[
I_{5\Omega} = \frac{4 \times 10}{10 + 5 + 3} = \frac{40}{18} = 2.22A
\]

19. Determine the current through 1Ω and 4Ω resistors. (JUN-09)

\[
I = \frac{10}{10} = 1A
\]
\[
I_{4\Omega} = \frac{1 \times 1}{5} = 0.2A
\]
\[
I_{1\Omega} = \frac{1 \times 4}{5} = 0.8A
\]
20. Find the current $I$ in the circuit. 
When 2A source is acting alone.

$$I_1 = \frac{2 \times 5}{10} = 1\text{A}$$
When 10V source is acting alone.

$$I_2 = \frac{10}{5 + 5} = 1\text{A}$$
When both sources are acting together $I=2\text{A}$

21. Find the thevenin’s equivalent for the circuit shown. (APR-08)

$$R_{th} = \frac{5 \times 5}{5 + 5} = 2.5\Omega$$
$$V_{th} = \frac{10 \times 5}{5 + 5} = 5\text{V}$$

22. Find the Norton’s equivalent for the circuit shown.

$$I_{sc} = \frac{10}{5} = 2\text{A}$$
$$R_{TH} = \frac{5 \times 10}{5 + 10} = 6.66\Omega$$

23. Find the value of $R_L$ for maximum power transfer.

Condition for maximum power transfer $R_L=R_{TH}$

$$R_{TH} = \frac{6 \times 6}{6 + 6} = 3\Omega$$
$$R_L = R_{TH} = 3\Omega$$

24. What is the property of additivity and homogeneity?

The principle of superposition is a combination of additivity and homogeneity.

The property of additivity says that the response in a circuit due to number of sources is given by sum of response due to individual sources acting alone.

The property of homogeneity says that if all the sources are multiplied by a constant, then the response is also multiplied by the same constant.
25. By using superposition theorem, find the current through the ammeter.

Response due to 10V source $I_1' = \frac{10}{4} = 2.5A$

Response due to 5V source $I_2' = -\frac{5}{2.5} = -2A$

Total response $= I = I_1' + I_2' = 2.5 + (-2) = 0.5A$

UNIT III
RESONANCE AND COUPLED CIRCUITS


3.6 SERIES AND PARALLEL RESONANCE THEIR FREQUENCY RESPONSE

Series Resonance

The basic series-resonant circuit is shown in fig. 1. Of interest here in how the steady state amplitude and the phase angle of the current vary with the frequency of the sinusoidal voltage source. As the frequency of the source changes, the maximum amplitude of the source voltage ($V_m$) is held constant.

$$V_s = V_m \cos(\omega t)$$

$$i = I_m \cos(\omega t + \theta)$$

The frequency at which the reactance of the inductance and the capacitance cancel each other is the resonant frequency (or the unity power factor frequency) of this circuit. This occurs at

$$\omega_0 = \frac{1}{\sqrt{LC}}$$  \hspace{1cm} (1)

Since $i = \frac{V_R}{R}$, then the current $i$ can be studied by studying the voltage across the resistor. The current $i$ has the expression

$$I_m = \frac{V_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$  \hspace{1cm} (2A)

$$\theta = -\tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$  \hspace{1cm} (2B)

The bandwidth of the series circuit is defined as the range of frequencies in which the amplitude of the current is equal to or greater than $(1/2 = 2/2)$ times its maximum amplitude, as shown in fig. 2. This yields the bandwidth $B = !2-!1 = R/L$
are called the half power frequencies or the 3 dB frequencies, i.e. the frequencies at which the value of \( I_m \) equals the maximum possible value divided by \( \frac{1}{\sqrt{2}} = 1.414 \). The quality factor

\[
Q = \frac{\omega_0}{B} = \frac{1}{R} \sqrt{\frac{L}{C}}
\]  

Then the maximum value of:

- VR occurs at \( \omega = \frac{\omega_0}{\sqrt{1 - \frac{R^2}{2L}}} \)  
- VL occurs at \( \omega = \frac{\omega_0}{\sqrt{1 - \frac{R^2}{2L}}} \)  
- VC occurs at \( \omega = \frac{\omega_0}{\sqrt{1 - \frac{R^2}{2L}}} \)

**Parallel Resonance:**

The basic parallel-resonant circuit is shown in fig. 3. Of interest here in how the steady state amplitude and the phase angle of the output voltage \( V_0 \) vary with the frequency of the sinusoidal voltage source.
The resonant frequency is
\[ \omega_0 = \frac{1}{\sqrt{LC}} \]

The 3 dB frequencies are:
\[ \omega_{2,1} = \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \pm \frac{1}{2RC} \] (7)

The bandwidth
\[ B = \omega_2 - \omega_1 = \frac{1}{RC}. \]

The quality factor
\[ Q = \frac{\omega_0}{B} = R \sqrt{\frac{C}{L}} \] (8)

3.7. QUALITY FACTOR AND BANDWIDTH

Quality factor, Q
Reactive components such as capacitors and inductors are often described with a figure of merit called Q. While it can be defined in many ways, it’s most fundamental description is:

\[ Q = \frac{\omega}{\frac{\text{energy stored}}{\text{average power dissipated}}} \]

Thus, it is a measure of the ratio of stored vs. lost energy per unit time. Note that this definition does not specify what type of system is required. Thus, it is quite general. Recall that an ideal reactive component (capacitor or inductor) stores energy

\[ E = \frac{1}{2} CV^2 \quad \text{or} \quad \frac{1}{2} LI^2 \]

Since any real component also has loss due to the resistive component, the average power dissipated is

\[ P_{\text{avg}} = \frac{1}{2} I_{pk}^2 R = \frac{V_{pk}^2}{2R} \]

If we consider an example of a series resonant circuit.

At resonance, the reactances cancel out leaving just a peak voltage, \( V_{pk} \), across the loss resistance, \( R \). Thus, \( I_{pk} = V_{pk}/R \) is the maximum current which passes through all elements. Then,

\[ Q = \frac{\omega \frac{LI_{pk}^2}{2}}{\frac{I_{pk}^2 R}{2}} = \frac{\omega L}{\omega R C} = \frac{1}{\omega R C} \]

In terms of the series equivalent network for a capacitor shown above, its Q is given by:

\[ Q = \frac{1}{\omega R C} = \frac{X}{R} \]

where we pretend that the capacitor is resonated with an ideal inductor at frequency \( \omega \). \( X \) is the capacitive reactance, and \( R \) is the series resistance. Since this Q refers only to the capacitor itself, in isolation from the rest of the circuit, it is called unloaded Q or \( Q_U \). The higher the unloaded Q, the lower the loss. Notice that the Q decreases with frequency.

The unloaded Q of an inductor is given by

\[ Q_U = \frac{\omega L}{R} \]

where \( R \) is a series resistance as described above. Note that Q is proportional to frequency for an inductor. The Q of an inductor will depend upon the wire diameter, core material (air, powdered iron, ferrite) and whether or not it is in a shielded metal can. It is easy to show that for a parallel resonant circuit, the Q is given by susceptance/conductance:

\[ Q = \frac{B}{G} \]

where B is the susceptance of the capacitor or inductor and G is the shunt conductance.

**Bandwidth:**

At a certain frequency the power dissipated by the resistor is half of the maximum power which as mentioned occurs at
\[ \omega_0 = \frac{1}{\sqrt{LC}}. \]

The half power occurs at the frequencies for which the amplitude of the voltage across the resistor becomes equal to \[ \frac{1}{\sqrt{2}} \] of the maximum.

\[ P_{1/2} = \frac{1}{4} \frac{V_{\text{max}}}{R} \]

Figure shows in graphical form the various frequencies of interest.

Therefore, the \( \frac{1}{2} \) power occurs at the frequencies for which

\[ \frac{1}{\sqrt{2}} = \frac{\omega_0}{\sqrt{\left(1 - \omega^2 LC\right)^2 + (\omega RC)^2}} \]

Equation has two roots

\[ \omega_1 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{\omega_0^2}} \]

\[ \omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{\omega_0^2}} \]

\[ \text{Bandwidth} = B = \omega_2 - \omega_1 \]

By multiplying these two Equations we can show that \( \omega_0 \) is the geometric mean of \( \omega_1 \) and \( \omega_2 \).

\[ \omega_0 = \sqrt{\omega_1 \omega_2} \]

As we see from the plot on Figure 2 the bandwidth increases with increasing \( R \). Equivalently the sharpness of the resonance increases with decreasing \( R \).

For a fixed \( L \) and \( C \), a decrease in \( R \) corresponds to a narrower resonance and thus a higher selectivity regarding the frequency range that can be passed by the circuit.

As we increase \( R \), the frequency range over which the dissipative characteristics dominate the behavior of the circuit increases. In order to quantify this behavior we define a parameter called the Quality Factor \( Q \) which is related to the sharpness of the peak and it is given by
\[ Q = 2\pi \frac{\text{maximum energy stored}}{\text{total energy lost per cycle at resonance}} = 2\pi \frac{E_s}{E_n} \]

Which represents the ratio of the energy stored to the energy dissipated in a circuit.

The energy stored in the circuit is

\[ E_s = \frac{1}{2} LI^2 + \frac{1}{2} CVc^2 \]

For \( Vc = A \sin\omega t \) the current flowing in the circuit is

\[ I = C \frac{dVc}{dt} = \omega CA \cos(\omega t) \]

The total energy stored in the reactive elements is

\[ E_s = \frac{1}{2} L\omega^2 C^2 A^2 \cos^2(\omega t) + \frac{1}{2} CA^2 \sin^2(\omega t) \]

At the resonance frequency where

\[ \omega = \omega_0 = \frac{1}{\sqrt{LC}} \]

the energy stored in the circuit becomes

\[ E_s = \frac{1}{2} CA^2 \]

The energy dissipated per period is equal to the average resistive power dissipated times the oscillation period.

\[ E_D = R \left( \langle I^2 \rangle \frac{2\pi}{\omega_0} \right) = R \left( \frac{\omega_0^2 C^2 A^2}{2} \right) \frac{2\pi}{\omega_0} = 2\pi \left( \frac{1}{2} \frac{RC}{\omega_0 L} A^2 \right) \]

And so the ratio \( Q \) becomes

By combining these Equations we obtain the relationship between the bandwidth and the Q factor.

\[ B = \frac{L}{R} = \frac{\omega_0}{Q} \]

**Summary of the properties of RLC resonant circuits.**
3.8. SELF AND MUTUAL INDUCTANCE

Self-Inductance:

Consider again a coil consisting of N turns and carrying current I in the counterclockwise direction, as shown in Figure. If the current is steady, then the magnetic flux through the loop will remain constant. However, suppose the current I changes with time, then according to Faraday’s law, an induced emf will arise to oppose the change. The induced current will flow clockwise if \( \frac{dI}{dt} > 0 \), and counterclockwise if \( \frac{dI}{dt} < 0 \). The property of the loop in which its own magnetic field opposes any change in current is called “self-inductance,” and the emf generated is called the self-induced emf or back emf, which we denote as \( L\varepsilon \). All current-carrying loops exhibit this property. In particular, an inductor is a circuit element (symbol ) which has a large self-inductance.
Mathematically, the self-induced emf can be written as
\[ \varepsilon_L = -N \frac{d \Phi_B}{dt} = -N \frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{A} \]
and is related to the self-inductance \( L \) by
\[ \varepsilon_L = -L \frac{dI}{dt} \]
The two expressions can be combined to yield
\[ L = \frac{N \Phi_B}{I} \]
Physically, the inductance \( L \) is a measure of an inductor’s “resistance” to the change of current; the larger the value of \( L \), the lower the rate of change of current.

**Mutual Inductance:**
Suppose two coils are placed near each other, as shown in Figure

Changing current in coil 1 produces changing magnetic flux in coil 2.
The first coil has \( N_1 \) turns and carries a current \( I_1 \) which gives rise to a magnetic field \( \mathbf{B}_1 \)... Since the two coils are close to each other, some of the magnetic field lines through coil 1 will also pass through coil 2. Let \( \Phi_{21} \) denote the magnetic flux through one turn of coil 2 due to \( I_1 \). Now, by varying \( I_1 \) with time, there will be an induced emf associated with the changing magnetic flux in the second coil:
\[ \varepsilon_{21} = -N_2 \frac{d \Phi_{21}}{dt} = - \frac{d}{dt} \int_{\text{coil 2}} \mathbf{B}_1 \cdot d\mathbf{A}_2 \]
The time rate of change of magnetic flux \( \Phi_{21} \) in coil 2 is proportional to the time rate of change of the current in coil 1:
where the proportionality constant $M_{21}$ is called the mutual inductance. It can also be written as

$$M_{21} = \frac{N_2 \Phi_{21}}{I_1}$$

The SI unit for inductance is the henry (H):

$$1 \text{ henry} = 1 \text{ H} = 1 \text{ T} \cdot \text{m}^2/\text{A}$$

We shall see that the mutual inductance $M_{21}$ depends only on the geometrical properties of the two coils such as the number of turns and the radii of the two coils.

In a similar manner, suppose instead there is a current $I_2$ in the second coil and it is varying with time. Then the induced emf in coil 1 becomes

$$\varepsilon_{12} = -N_1 \frac{d\Phi_{12}}{dt} = -\frac{d}{dt} \iint_{\text{coil 1}} \mathbf{B}_2 \cdot d\mathbf{A}_1$$

and a current is induced in coil 1.

Changing current in coil 2 produces changing magnetic flux in coil 1. This changing flux in coil 1 is proportional to the changing current in coil 2

$$N_1 \frac{d\Phi_{12}}{dt} = M_{12} \frac{dI_2}{dt}$$

where the proportionality constant $M_{21}$ is another mutual inductance and can be written as

$$M_{12} = \frac{N_1 \Phi_{12}}{I_2}$$

However, using the reciprocity theorem which combines Ampere’s law and the Biot-Savart law, one may show that the constants are equal:

$$M_{12} = M_{21} = M$$

### 3.9. COEFFICIENT OF COUPLING

In coupled coils, the coefficient of coupling is defined as the fraction of the total flux produced by one coil linking another coil.
Coefficient of coupling $K = \frac{\varphi_{12}}{\varphi_1} = \frac{\varphi_{21}}{\varphi_2}$

3.10. TUNED CIRCUITS

Many communication applications use tuned circuits. These circuits are assembled from passive components (that is, they require no power supply) in such a way that they only respond to a narrow band of frequencies.

Applications include:
- Radio Receivers - RF Amplifier, Local Oscillator, IF Amplifier
- Filters for frequency division multiplexing - reception filters.
- Filters to restrict bandwidth of a signal prior to transmission.
- General band-pass and band-stop filters.

A tuned circuit passes or rejects all frequencies except those grouped around the resonant frequency of the circuit. Both the resonant frequency, and the spread of frequencies transmitted (bandwidth), are dependent on the values of the components used to make up the tuned circuit.

Passive tuned circuits contain three basic components;  
3.1. inductors,  
3.2. capacitors and  
3.3. Resistors.

Review:

4.1. The reactance of an inductor is proportional to the frequency (f) of the current flowing through it, so with increasing frequency the reactance/impedance of the component increases. $\text{XL} = 2\pi f L$

4.2. The reactance of a capacitor is inversely proportional to the frequency (f) of the current flowing through it, so with increasing frequency the reactance/impedance of the component decreases. $\text{XC} = \frac{1}{2\pi f C}$

4.3. When used together an inductor and capacitor become a resonant circuit.

4.4. Resonance occurs when $\text{XL} = \text{XC}$ ($2\pi f L = 1/2 f C$)

\[ \text{Resonant Frequency } f_0 = \frac{1}{2\pi\sqrt{LC}} \]

- Single tuned circuits.

Another parameter of a tuned circuit is the Bandwidth; this is determined by the quality (Q) of the circuit. The bandwidth is the frequency difference between the lower and upper 70% maximum amplitude (-3dB) points of the tuned circuits response curve.

\[ \text{Bandwidth (BW)} = \frac{f_0}{Q} \quad \therefore Q = \frac{f_0}{\text{BW}} \]

LC Tuned Circuit – Bandwidth and ‘Q’ (2)
In the LC resonant circuit the Q of the circuit is determined by the inductor. The ideal inductor is a pure reactance, however a practical inductor, which is a long length of wire wound round a magnetic material, has a finite resistance.

The Q of an inductor is given by:

\[ Q = \frac{2\pi f_0 L}{R_0} \]

The Q of an inductor is usually specified at a particular frequency, so you have to calculate what it will be at any other frequency. Since a large Q results in a small bandwidth when the inductor is used as part of a tuned circuit, it obviously pays to use a large inductor at all times. Narrow bandwidths are going to be easier to generate at high frequencies than low ones. For this reason, tuned circuits tend to be fairly useless as narrow bandwidth filters below 1MHz.

**LC Parallel Circuit**

This circuit is used as the tuned element in many communications transmitters and receivers. To illustrate the impedance characteristics and Current/Voltage flow we will consider the parallel LC circuit connected in series with a fixed resistor, as detailed opposite:

Below the resonant frequency (at lower frequencies): \( XL << XC \) The impedance of the parallel combination is dependant mainly on the inductor (L), which has a low impedance. The capacitor has a very high impedance compared to that of the inductor.

The current mainly flows through the inductor, and the volt-drop across the LC circuit is low, most of the voltage being dropped across the resistor.

**LC Parallel Circuit (as the basic tuner in a radio receiver)**

It has been shown that if the LC circuit is connected in series with a fixed component it can be used to select (and give an output) only at a given frequency (the resonant frequency).

In a radio receiver, a parallel tuned circuit is connected to the antenna/aerial system - a signal voltage is only developed across the LC circuit when a radio signal is received at the resonant frequency. By adjusting the capacitor, the tuned frequency of the circuit can be changed to allow another radio station to be received.
LC Parallel Circuit ‘Q’ and Selectivity:

The ‘Q’ of the inductor determines the bandwidth of the response peak:

1. The lower the Q, the greater the bandwidth, and the less selective the circuit.
2. The quality factor (Q) of the LC circuit(s) used in a radio receiver therefore determines its selectivity; that is, how well is can tune to and select a single broadcast channel from a given waveband.
QUESTION BANK
PART-A
TWO MARKS

1. Define resonance. What is the condition for resonance for an RLC series circuit?
[MAY/JUNE’07] (JUN-09) [DEC-10]
A circuit is said to be in resonance when the applied voltage and current are in phase. For an RLC series circuit, at resonance the inductive and capacitive reactance are equal.

2. How the RLC series circuit behaves for the frequencies above and below the resonant frequencies.
For frequencies below resonant frequency, the capacitive reactance is more than the inductive reactance. Therefore the equivalent reactance is equal to capacitive and the circuit behaves like a RC circuit.
For frequencies above resonant frequency, the inductive reactance is more than the capacitive reactance. Therefore the equivalent reactance is equal to inductive and the circuit behaves like a RL circuit.

3. Derive the expression for resonant frequency. [MAY/JUNE’07] (JUN-12)
At resonance condition, the inductive and capacitive reactances are equal.
\[ X_L = X_C \]
\[ 2\pi f L = \frac{1}{2\pi f C} \]
\[ f^2 = \frac{1}{4\pi^2 LC} \]
\[ f_r = f_o = \frac{1}{2\pi \sqrt{LC}} \]

4. Define resonant frequency. [MAY/JUNE’05] (JUN-08)
The frequency at resonance is called as resonant frequency.
It is also defined as the geometric mean of two half power frequencies is called resonant frequency.
\[ f_o = \sqrt{f_1f_2} \]

5. Draw the phasor diagrams for series RLC circuit.

6. Draw the curves for variation of impedance, admittance and current with frequency in RLC series resonance circuit.
7. Define Q factor. [NOV/DEC’03] (JUN-09) [MAY-11]
   It is the ratio between capacitor voltage or inductor voltage at resonance to supply voltage is called as Q-factor or quality factor.
   \[ Q \text{ factor} = \frac{\text{capacitor voltage or inductor voltage}}{\text{supply voltage}} \]
   It is also defined as
   \[ Q \text{ factor} = 2\pi \times \frac{\text{Maximum energy stored}}{\text{Energy dissipated per cycle}} \]
   \[ Q \text{ factor} = \frac{\omega L}{R} = \frac{1}{\omega RC} = \frac{1}{R \sqrt{C}} = \frac{f_0}{f_0} \]

8. Define Bandwidth. (JUN-09)
   It is defined as the width of the resonant curve upto frequency at which the power in the circuit is half of its maximum value. The difference between two half power frequencies is also called as bandwidth.

   It is the ratio of bandwidth to resonant frequency.
   \[ \text{Selectivity} = \frac{\text{Bandwidth}}{\text{Resonant frequency}} = \frac{f_2 - f_1}{f_0} \]
   Selectivity of a resonant circuit is its ability to discriminate between signals of desired and undesired frequencies.

10. Define half power frequencies.

   The frequencies at which the power in the circuit is half of its maximum value are called as half power frequencies.
   \[ f_1 = \text{lower cut-off frequency} \]
   \[ f_2 = \text{upper cut-off frequency} \]

11. Give the relationship between
   a) Band width, resonant frequency and quality factor
   \[ Q_0 = \frac{f_0}{f_2 - f_1} = \frac{\omega L}{R} \]
   b) Resonant frequency and half power frequencies
   \[ \omega_o = \sqrt{\omega_1 \omega_2} \]
   c) Quality factor in terms of R, L and C
12. In series resonance circuit, on resonance the following will occur
   a) \( V = V_R \)
   b) \( X_L = X_C \)
   c) \( Z = R \)
   d) \( V_L = V_C \)
   e) all the above
   ANS: all the above

13. What inductance will give the same reactance as a capacitor of 2μF when both are at 50Hz?
   The inductive and capacitive reactances are same
   \[ X_L = X_C \]
   \[ 2\pi f L = \frac{1}{2\pi f C} \]
   \[ 4\pi^2 f^2 L = \frac{1}{C} \]
   \[ L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2 \left( \frac{1}{C} \right) \times 10^{-6}} = 5H \]

14. A series resonant circuit is capacitive at \( f = 150 \text{ Hz} \), The circuit will be inductive some where at
   a) \( f > 150\text{Hz} \)
   b) \( f = 150\text{Hz} \)
   c) \( f < 150\text{Hz} \)
   d) None of the above
   ANS: \( f > 150\text{Hz} \)

15. A series RLC circuit has \( R = 50\Omega; L = 100\mu\text{H}; \text{and } C = 300\text{pF}, v = 20\text{V}. \) What is the current at resonance?
   \[ I = \frac{V}{R} = \frac{20}{50} = 0.4\, \text{A} \]

16. An RLC circuit consists of a resistance of 1000Ω, an inductance of 100 mH and a capacitance of 10μF. The Q factor of the circuit is
   \[ Q = \frac{1}{R} \sqrt{\frac{L}{\frac{L}{C}}} = \frac{1}{1000} \sqrt{\frac{100 \times 10^{-3}}{10 \times 10^{-6}}} = 0.1 \]

17. Write the characteristics of series resonance.
   A series RLC circuit, at resonance condition
   i) The power factor is unity
   ii) Impedance of the circuit is minimum
   iii) Admittance of the circuit is maximum
   iv) Current is maximum
   v) The magnitude of the voltage across inductance and capacitance will be \( Q \) times the supply voltage, but they are in phase opposition.

18. Write the expression for half power frequencies of RLC series circuit.
Lower cut-off frequency \( f_1 = \frac{1}{2\pi} \left[ -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \right] \) or \( \left[ -\frac{1}{2Q_r} + \sqrt{1 + \frac{1}{4Q_r^2}} \right] \)

Upper cut-off frequency \( f_2 = \frac{1}{2\pi} \left[ \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \right] \) or \( \left[ \frac{1}{2Q_r} + \sqrt{1 + \frac{1}{4Q_r^2}} \right] \)

19. An RLC series circuit has \( R = 10\Omega, X_C = 62.833\,\Omega \). Find the value of \( L \) for resonance at 50HZ.

At resonance, \( X_L = X_C \)

\[ X_L = 2\pi f L, \text{ Inductance, } L = \frac{X_L}{2\pi f} = \frac{62.833}{2\pi \times 50} = 0.2\,H \]

20. Determine the quality factor of the RLC series circuit with \( R = 10\Omega, L = 0.01\,H \) and \( c = 100\mu F \).

Quality factor at resonance \( Q_r = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{1}{10} \sqrt{\frac{0.01}{100 \times 10^{-6}}} = 1 \)

21. A series RLC circuit is excited with 10V sinusoidal source resonate at a frequency of 50HZ. If the bandwidth is 5HZ. What will the voltage across capacitance?

Quality factor at resonance \( Q_r = \frac{f_0}{B.W} = \frac{50}{5} = 10 \)

Voltage across capacitor \( Q_r V = 10 \times 10 = 100V \)

22. A series RLC circuit has \( R = 10\Omega, X_C = 20\,\Omega \) and \( X_L = 20\,\Omega \) is excited by a sinusoidal source of voltage 200V. What will the voltage across inductance.

Quality factor at resonance \( Q_r = \frac{\omega_0 L}{R} = \frac{X_L}{R} = \frac{20}{10} = 2 \)

Voltage across inductor \( Q_r V = 2 \times 200 = 400V \)

23. The impedance and quality factor of a RLC series circuit at \( \omega_0 = 10^7 \) rad/sec are 100+j0 and 100 respectively. Find the values of \( R, L \) and \( C \).

Given

\[ \omega_0 = 10^7 \text{ rad/sec} \]
\[ \text{Impedance} = 100+j0 \]
\[ \text{Quality factor} = 100 \]

At resonance \( Z = R \), Resistance = 100\Omega

Quality factor \( Q_r = \frac{\omega_0 L}{R} \)

Inductance \( L = \frac{Q_r R}{\omega_0} = \frac{100 \times 100}{10^7} = 1mH \)

\[ \omega_0^2 = \frac{1}{LC} \]

\[ C = \frac{1}{\omega_0^2 L} = \frac{1}{(10^7)^2 \times 1 \times 10^{-3}} = 10\,pF \]
24. What is anti-resonance?
In RLC parallel circuit, the current is minimum at resonance whereas in series resonance the current is maximum. Therefore the parallel resonance is called anti-resonance.

25. Write the expressions for quality factor of parallel RLC circuit.
\[ Q \text{ factor} = \frac{R}{\omega C} = \frac{1}{\sqrt{LC}} = \frac{f_0}{B.W} \]

26. Write the expression for half power frequencies of parallel RLC circuit.
Lower cut-off frequency \( f_1 = \frac{1}{2\pi} \left[ \frac{1}{RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \right] \) or \( -\frac{1}{2Q_r} + \sqrt{1 + \frac{1}{4Q_r^2}} \)
Upper cut-off frequency \( f_2 = \frac{1}{2\pi} \left[ \frac{1}{RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \right] \) or \( \frac{1}{2Q_r} + \sqrt{1 + \frac{1}{4Q_r^2}} \)

27. What is dynamic resistance? Write the expression for dynamic resistance of RL circuit parallel with C.
The resistance of the RLC parallel circuit at resonance is called dynamic resistance.
\[ R_{\text{dynamic}} = \frac{L}{CR} \]

28. Write the characteristics of parallel resonance.
   i) At resonance, admittance is minimum and equal to conductance, therefore current is minimum.
   ii) Below resonant frequency, the circuit behaves as inductive circuit and above resonant frequency, the circuit behaves as capacitive circuit.
   iii) At resonance, the magnitude of current through inductance and capacitance will be \( Q \) times the current supplied by the source, but they are in phase opposition.

29. Draw the curves for variation of impedance, admittance and current with frequency in RLC parallel resonance circuit.

30. Compare the series and parallel resonant circuit (consider practical parallel resonant circuit)
### Table: Comparison of Series and Parallel RLC Circuits

<table>
<thead>
<tr>
<th>Description</th>
<th>Series RLC circuit</th>
<th>Parallel RLC circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impedance at resonance</td>
<td>Minimum $\frac{V}{R}$</td>
<td>Maximum $\frac{V}{CR}$</td>
</tr>
<tr>
<td>Current at resonance</td>
<td>Maximum $\frac{V}{R}$</td>
<td>Minimum $\frac{V}{CR}$</td>
</tr>
<tr>
<td>Effective impedance</td>
<td>$R$</td>
<td>$\frac{L}{CR}$</td>
</tr>
<tr>
<td>Power factor at resonance</td>
<td>Unity</td>
<td>Unity</td>
</tr>
<tr>
<td>Resonant frequency</td>
<td>$\frac{1}{2\pi\sqrt{LC}}$</td>
<td>$\frac{1}{2\pi\sqrt{1 - \frac{R^2}{L^2}}}$</td>
</tr>
<tr>
<td>It magnifies</td>
<td>Voltage</td>
<td>Current</td>
</tr>
<tr>
<td>Magnification is</td>
<td>$\frac{\omega L}{R}$</td>
<td>$\frac{\omega L}{R}$</td>
</tr>
</tbody>
</table>

31. **Why is a series resonance circuit regarded as acceptor circuit?**

A series resonance circuit has a capability to draw heavy currents and power from the mains. So it is regarded as acceptor circuit.

32. **Why is a parallel resonance circuit regarded as rejector circuit?**

A parallel resonance circuit has a capability to very small currents and power from the mains. So it is regarded as rejector circuit.

33. **A coil of resistance $R$, an inductance $L$ is shunted by a capacitor $C$ and this parallel combination is under resonance. What is the resonate frequency of the circuit?**

![Parallel RLC Circuit Diagram](image)

Resonant frequency $f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}$

34. **Find the resonant frequency in the ideal parallel RLC circuit with $L=40$ mH and $C=0.01\mu F$.**

Resonant frequency $f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{40 \times 10^{-3} \times 0.01 \times 10^{-6}}} = 7958\text{HZ}$

35. **Draw the two branch parallel resonant circuit.**

Resonant frequency $f_0 = \frac{1}{2\pi\sqrt{LC}} \times \sqrt{\left(\frac{R_1^2 - \frac{L}{C}}{R_2^2 - \frac{L}{C}}\right)}$

36. **What is the nature of power factor if the frequency of operation is less than the resonance frequency in a RLC series circuit?**
When the frequency is less than resonance frequency \( f_0 \), the current lags behind the voltage. i.e., \( \phi \) is leading in nature. Hence, the power factor is leading for frequency less than the resonance frequency.

37. Draw the variation of reactance with frequency in series resonance circuit.

38. Draw the variation of susceptance with frequency in parallel resonance circuit.

39. A series RLC circuit has a bandwidth of 500 HZ. The quality factor is 15. If the value of L is 0.02, find the value of C.

\[
fo = \frac{1}{2\pi \sqrt{LC}}
\]

\[
fo = Bandwidth \times Quality\ factor
\]

\[
500 \times 15 = 7500 HZ
\]

\[
C = \frac{1}{4\pi^2 \times 7500^2 \times 0.02} = 5 \mu F
\]

40. What is magnetic coupling?

When two coils are placed close to each other, magnetic flux produced in one coil will be linking partially or fully with the other coil. This is called magnetic coupling.

41. Define induced emf.

It is the property of electromagnetic fields that whenever a coil has an alternating current passed through it, an emf is induced in it due to the alternating magnetic field surrounding the coil. Types of induced emf:

(i) Self induced emf
(ii) Mutually induced emf

42. Define self induced emf.

When an alternating current is passed through a coil, an alternating magnetic field is set up, which surrounds the coil. This induced emf is called self induced emf. It is so called because emf is induced due to its own magnetic field.

43. Define mutually induced emf.

When two coils are placed very close to each other, an emf is induced in the other coil also, which is called mutually induced emf.
44. Define inductance.
   It is defined as the flux linkages per unit current. There are two types of inductance.
   (i) Self inductance
   (ii) Mutual inductance

45. Define self inductance. (JUN-09)
   Self inductance of a coil is defined as the flux linkages per unit current flowing through the
   coil. Its unit is Henry.
   \[ L = \frac{N\phi}{I} \]
   The self induced emf in a coil can be expressed in terms of self inductance.
   \[ e = -L\frac{di}{dt} \]

46. Define mutual inductance. [MAY/JUNE’07] (JUN-08)
   Mutual inductance between two coils is defined as the flux linkages in one coil due to unit
   current in other coil. Its unit is Henry.
   \[ M = \frac{N_2\phi_1}{I_1} \]
   \[ M = \frac{N_1\phi_2}{I_2} \]
   Induced emf in a coil can be expressed in terms of mutual inductance.
   \[ e_2 = -M\frac{di_1}{dt} \]
   \[ e_1 = -M\frac{di_2}{dt} \]
   \( e_1 \) is the induced emf in coil 1 due to a current \( i_2 \) flowing in coil 2.

47. What are coupled circuits?
   The coupled coils refer to circuits involving elements with magnetic coupling. If the flux
   produced by an element of a circuit links other elements of the same circuit or near-by circuit then the
   elements are said to have magnetic coupling.

48. What are coupled coils?
   When two or more coils are linked by magnetic flux. then the coils are called coupled coils(or
   the coupled coils are group of two or more coils linked by magnetic flux).

49. Define coefficient of coupling. [NOV/DEC’07] [NOV/DEC’04] [MAJ/JUNE’05]
[MAY/JUNE’07] (JUN-09)[DEC-10]
   In coupled coils, the coefficient of coupling is defined as then fraction of the total flux
   produced by one coil linking another coil.
   \[ \text{Coefficient of coupling} = K = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2} \]

50. What is dot convention? Why it is required?
   The sign or polarity of mutually induced emf depends on the winding sense (or coil
   orientation) and current through the coil. The winding sense is decided by the manufacturer and to
   inform the user about the winding sense a dot is placed at one end/terminal of each coil. When current
   enter at dotted end in one coil then the mutual induced emf in the other coil is positive at dot end.
   Hence the dot convention is required to predict the sign of mutually induced emf.

51. State Dot rule for coupled circuits.
If current enters into the dots of both the coils or out of dots of both the coils, then the mutually induced voltages for both the coils are having the same polarity of self induced voltages.

If current enter into (or out of) the dot in one coil and in other coil current flows out of (or into) the dot, then the mutually induced voltage will have polarity opposite to that of self induced voltages.

52. Two coupled coils have a self inductances 37.5 mH and 193 mH, with the mutual inductance of 63.75 mH. Find the coefficient of coupling.

\[ K = \frac{M}{\sqrt{L_1 L_2}} = \frac{63.75}{\sqrt{37.5 \times 193}} = 0.75 \]

54. A 15 mH coil is connected in series with another coil. The total inductance is 70 mH. When one of the coils is reversed, the total inductance is 30 mH. Find the self inductance of second coil, mutual inductance and coefficient of coupling.

Given:

Self inductance of coil 1 = \( L_1 = 15 \) mH
Series aiding \( L_{eq} = L_1 + L_2 + 2M = 70 \) mH
Series opposing \( L_{eq} = L_1 + L_2 - 2M = 30 \) mH

From above equations
\[ 15 + L_2 + 2M = 70 \]
\[ 15 + L_2 - 2M = 30 \]
\[ L_2 + 2M = 55 \]
\[ L_2 - 2M = 15 \]

From above equations
\[ L_2 = 35 \text{ mH} \]
\[ 35 + 2M = 55 \]
\[ M = 10 \text{ mH} \]

55. Write the expression for equivalent inductance of two coupled coils connected in series.

In series aiding \( L_{eq} = L_1 + L_2 + 2M \)
In series opposing \( L_{eq} = L_1 + L_2 - 2M \)

56. Write the expression for equivalent inductance of two coupled coils connected in parallel.

Parallel aiding \( L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} \)
Parallel opposing \( L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} \)

57. Write the relation between self and mutual inductance

\[ M = K \sqrt{L_1 L_2} \]
\( L_1 = \) Self inductance of coil 1
\( L_2 = \) Self inductance of coil 2
\( K = \) Coefficient of coupling

58. What is the maximum possible mutual inductance of two inductively coupled coils with self inductances of 50 mH and 200 mH?

\( L_1 = 50 \) mH
\( L_2 = 200 \) mH
\( K = 1 \) for max \( M \)
\[ M = K \sqrt{L_1 L_2} \]
59. Find the equivalent inductance of the circuit shown.
\[ Leq = L_1 + L_2 + L_3 + 2M_{12} - 2M_{13} - 2M_{23} \]
\[ = 6 + 4 + 2 + 2 - 2 - 2 - 2 \times 0.5 \]
\[ = 6 + 4 + 2 - 4 - 1 \]
\[ = 9 \text{ H} \]

60. What is tuned coupled circuit?
In a coupled circuit, when capacitors are added to primary and secondary of coupled coils to resonate the coils to achieve maximum power transfer condition, then the coupled circuit is called tuned coupled circuit.

61. Why and where the tuned coupled circuits are employed?
The tuned coupled circuits are mainly used to transfer energy from a weak source to a load or employed for maximum power transfer from one circuit to another circuit. This is possible only when both coils work at resonance condition.

62. What is single tuned circuit?
In a coupled circuit, when capacitors are added to secondary coil to resonate the secondary, the coupled circuit is called single tuned coupled circuit.

63. What is double tuned circuit?
In a coupled circuit, when capacitors are added both primary and secondary coils to resonate the primary and secondary, the coupled circuit is called double tuned coupled circuit.

PART-B
16 MARKS

1. Explain in details about the step response of series RLC circuit.[AU-MAY-11][JUN-12]

In series RLC circuit, there are two energy storing element which are L and C, such a circuit give rise to second order differential equation and hence called second order circuit.
Consider a series RLC circuit shown in Figure. The switch is closed at \( t = 0 \) and a step voltage of \( V \) volts gets applied to circuit.

Apply KVL after switching we get
\[ L \frac{di}{dt} + Ri + \frac{1}{C} \int i dt = V \]
As ‘V’ is step i.e., constant, differentiating both sides of above equation gives,
\[ \frac{Ld^2i}{dt^2} + \frac{Rdi}{dt} + \frac{i}{C} = 0 \]
\[ \frac{d^2i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{1}{CL} i = 0 \]
\[
Si^2 - \frac{R}{L} Si + \frac{1}{LC} i = 0
\]

Where \( S = \frac{d}{dt} \)

This is called characteristic equation or auxiliary equation of series RLC circuits.
The response of the circuit depends on the nature of the roots of characteristic equation. The two roots are,

\[
S_{1,2} = \frac{-R}{L} \pm \frac{\sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2} = \frac{-R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}
\]

Let us define some quantities to find the response according to the nature of roots.

**Critical resistance :-**
This is the value of resistance which reduces square root term to zero, giving real, equal and negative roots.

\[
\frac{R_{cr}}{2L} = \frac{1}{\sqrt{LC}}
\]

\[
R_{cr} = 2\sqrt{\frac{L}{C}}
\]

**Damping Ratio (\(\xi\)):-**
The ratio is the indication of opposition from the circuit to cause oscillations in its response more the value of this ratio, less are the chances of oscillations in the response. It is the ratio of actual resistance in the circuit to critical resistance.

It is denoted by zeta(\(\xi\)),

\[
\xi = \frac{R}{R_{cr}} = \frac{R}{2\sqrt{\frac{L}{C}}}
\]

3. **Natural frequency (\(\omega_n\)):-**
If the damping is made zero then the response oscillates with natural frequency without any opposition. Such a frequency when \(\xi = 0\) is called natural Frequency of oscillations, denoted as \(\omega_n\). It is given by

\[
\omega_n = \frac{1}{\sqrt{LC}}
\]

Using these values, the roots of equation are

\[
S_{1,2} = -\xi \omega_n \pm \omega_n \sqrt{\xi^2 - 1}
\]

\[
= \xi \omega_n \pm j \omega_n \sqrt{1-\xi^2}
\]

Thus the response is totally dependent on the values of \(\xi\).
Let \(\alpha = \xi \omega_n\)
\[ w_d = \sqrt{1 - \xi^2} \]

Where \( w_d \) = actual frequency of oscillations (i.e.) damped frequency when \( \xi = 0 \), we get \( w_d = \omega_n \) (i.e.) natural frequency.

The general solution of characteristic equation is

\[ iC \frac{d}{dt} = K_1 e^{(-\alpha+jw_d)t} + K_2 e^{(-\alpha-jw_d)t} \]

2. Define Q-factor; Derive the expressions for Q-factor of inductor and capacitor.

Write a note on figure of merit.

The resonance phenomenon is observed in ac circuits consisting reactive elements such as inductor and capacitor. These two elements are basic passive elements of energy storing type.

It is found convenient to express the efficiency with which these elements store energy. It is also found simpler to compare various inductors and capacitors in terms of efficiency while designing such circuits. Such efficiency is measured as quality factor. It is also called figure of merit.

The figure of merit or Q-factor is defined as

\[ Q = 2\pi \frac{\text{Maximum energy stored per cycle}}{\text{Energy dissipated per cycle}} \]

Let us derive the expressions for the quality factor of inductor and capacitor.

Consider that sinusoidal voltage \( V \) is applied to an inductor with leakage resistance \( R \) in series as shown in fig.

The maximum energy stored per cycle is given by

\[ E_L = \frac{1}{2} L I_m^2 \]

Where \( I_m \) = Peak or maximum value of current in the circuit.

The average power dissipated in inductor is given by

\[ P_D = \left( \frac{I_m}{\sqrt{2}} \right)^2 R = \frac{1}{2} I_m^2 R \]

The total energy dissipated / cycle is given by

\[ P_D = \frac{1}{2} I_m^2 R \]

Energy dissipated / cycle = \( \frac{P_D}{f} \)

Where, \( f \) = frequency of operation.

Thus by definition, the quality factor of inductor is given by,
Let us consider that sinusoidal voltage $V$ is applied to a capacitor with a small resistance $R$ in series as shown in Fig. 1.

The maximum energy stored per cycle is given by

$$EC = \frac{1}{2} CV_m^2$$

where $V_m =$ peak or maximum value of applied voltage

$$= \frac{1}{2} C \left( \frac{I_m}{\omega C} \right)^2 = \frac{1}{2} \frac{I_m^2}{\omega^2 C}$$

The average power dissipated in the capacitor is given by

$$PD = \left( \frac{I_m}{\sqrt{2}} \right)^2 R = \frac{1}{2} I_m^2 R$$

The total energy dissipated per cycle is given by,

Energy dissipated /cycle =

$$\frac{PD}{f} = \frac{1}{2} \frac{I_m^2}{f}$$

where

$f =$ frequency of operation,

Thus by definition, the quality factor of the capacitor is given by,

$$Q = 2\pi \times \frac{1}{2} \frac{I_m^2}{\frac{1}{2} I_m^2 R} \frac{1}{\omega^2 R}$$

$$Q = \frac{2\pi f}{\omega^2 RC}$$

$$Q = \frac{1}{\omega RC}$$

3). Write a note on linear transformer.
An alternating voltage can be raised or lowered as per requirements in the different stages of electrical network by using static device called as transformer.

The transformer works on the principle of mutual induction. It transfers electric energy from one circuit to another when there is no electrical connection between the two circuits.

The transformer is a static device in which electric power is transformed from one alternating current circuit to another with desired change in voltage and current, without any change in frequency.

The principle of mutual induction states that when two coils are inductively coupled and if the current in one coil changed uniformly then an e.m.f gets induced in the other coil. This e.m.f can drive a current, when a closed path is provided to it. The transformer works on the same principle. In its elementary form, it consists of two inductive coils which are electrically separated but linked through a common magnetic circuit. The two coils have high mutual inductance. The basic transformer is shown in fig.

One of the two coils is connected to a source of alternating voltage. This coil in which electrical energy is fed with the help of source called primary winding (P). The other winding is connected to the load. The electrical energy transformed to this winding is drawn out to the load. This winding is called secondary winding (S). The primary winding has \(N_1\) number of turns while the secondary winding has \(N_2\) number of turns. Symbolically the transformer is indicated as shown in fig.

When the primary winding is exited by an alternating voltage, it circulates an alternating current. This current produces an alternating flux (\(\Phi\)) which completes its path through common magnetic core as shown dotted in fig, thus an alternating flux links with secondary winding. As the flux is alternating, according to Faraday's law of electromagnetic induction, mutually induced e.m.f gets developed in the secondary winding. If now the load is connected to the secondary winding, this e.m.f drives a current through it.

Thus though there is no electrical contact between the two windings, an electrical energy gets transferred from primary to the secondary.

4. A pure inductance of 150 mH is connected in parallel with a 40 \(\mu\)F capacitor across a 50 V, variable frequency supply. Determine (a) the resonant frequency of the circuit and (b) the current circulating in the capacitor and inductance at resonance.
The circuit diagram is shown in Figure 16.10.

(a) Parallel resonant frequency, \( f_r = \frac{1}{2\pi} \left( \frac{1}{LC} - \frac{R^2}{L^2} \right) \)

However, resistance \( R = 0 \). Hence,

\[
fr = \frac{1}{2\pi} \sqrt{\frac{1}{LC}} = \frac{1}{2\pi} \sqrt{\frac{1}{150 \times 10^{-3} \times 40 \times 10^{-6}}} = \frac{10^3}{2\pi} \left( \frac{1}{6} \right) = 64.97 \text{ Hz}
\]

(b) Current circulating in L and C at resonance,

\[
ICIRC = \frac{V}{XC} = \frac{V}{\frac{1}{2\pi f_r C}} = 2\pi f_r CV
\]

Hence \( ICIRC = 2\pi 64.97 \times 40 \times 10^{-6} \times 50 = 0.816 \text{ A} \)

Alternatively, \( ICIRC = \frac{V}{XL} = \frac{V}{2\pi f_r L} = \frac{50}{2\pi 64.9 \times 0.15} = 0.817 \text{ A} \)

1- A series L–R–C circuit has a sinusoidal input voltage of maximum value 12 V. If inductance, \( L = 20 \text{ mH} \), resistance, \( R = 80 \Omega \), and capacitance, \( C = 400 \text{ nF} \), determine (a) the resonant frequency, (b) the value of the p.d. across the capacitor at the resonant frequency, (c) the frequency at which the p.d. across the capacitor is a maximum, and (d) the value of the maximum voltage across the capacitor.[AU-DEC-10]
(a) The resonant frequency,

\[ f_r = \frac{1}{2\pi \sqrt{LC}} = \frac{1}{2\pi \sqrt{[(20 \times 10^{-3})(400 \times 10^{-9})]}} \]

\[ = 1779.4 \text{ Hz} \]

(b) \( V_C = QV \) and \( Q = \frac{\omega L}{R} \left( \text{or} \frac{1}{\omega CR} \text{ or } \frac{1}{R \sqrt{C}} \right) \)

Hence \( Q = \frac{(2\pi 1779.4)(20 \times 10^{-3})}{80} = 2.80 \)

Thus \( V_C = QV = (2.80)(12) = 33.60 \text{ V} \)

(c) From equation (28.7), the frequency \( f \) at which \( V_C \) is a maximum value,

\[ f = f_r \sqrt{\left(1 - \frac{1}{2Q^2}\right)} = (1779.4) \sqrt{\left(1 - \frac{1}{2(2.80)^2}\right)} \]

\[ = 1721.7 \text{ Hz} \]

(d) From equation (28.9), the maximum value of the p.d. across the capacitor is given by:

\[ V_{Cm} = \frac{QV}{\sqrt{\left[1 - \left(\frac{1}{2Q}\right)^2\right]}} = \frac{(2.80)(12)}{\sqrt{\left[1 - \left(\frac{1}{2(2.80)}\right)^2\right]}} = 34.15 \text{ V} \]

2- A coil of inductance 5 mH and resistance 10 Ω is connected in parallel with a 250 nF capacitor across a 50 V variable-frequency supply. Determine (a) the resonant frequency, (b) the dynamic resistance, (c) the current at resonance, and (d) the circuit Q-factor at resonance.
(a) Resonance frequency

\[ f_r = \frac{1}{2\pi} \sqrt{\left( \frac{1}{LC} - \frac{R^2}{L^2} \right)} \text{ from equation (29.3),} \]

\[ = \frac{1}{2\pi} \sqrt{\left( \frac{1}{5 \times 10^{-3} \times 250 \times 10^{-9}} - \frac{10^2}{(5 \times 10^{-3})^2} \right)} \]

\[ = \frac{1}{2\pi} \sqrt{(800 \times 10^6 - 4 \times 10^6)} = \frac{1}{2\pi} \sqrt{(796 \times 10^6)} = 4490 \text{ Hz} \]

(b) From equation (29.4), dynamic resistance,

\[ R_D = \frac{L}{CR} = \frac{5 \times 10^{-3}}{(250 \times 10^{-9})(10)} = 2000 \Omega \]

(c) Current at resonance, \[ I_r = \frac{V}{R_D} = \frac{50}{2000} = 25 \text{ mA} \]

(d) Q-factor at resonance, \[ Q_r = \frac{\omega_r L}{R} = \frac{(2\pi 4490)(5 \times 10^{-3})}{10} = 14.1 \]
UNIT IV  
TRANSIENT RESPONSE FOR DC CIRCUITS

Transient response of RL, RC and RLC Circuits using Laplace transform for DC input and A.C. input with sinusoidal input – Characterization of two port networks in terms of Z, Y and h parameters.

4.7. INTRODUCTION:

For higher order differential equation, the number of arbitrary constants equals the order of the equation. If these unknowns are to be evaluated for particular solution, other conditions in network must be known. A set of simultaneous equations must be formed containing general solution and some other equations to match number of unknown with equations.

We assume that at reference time $t=0$, network condition is changed by switching action. Assume that switch operates in zero time. The network conditions at this instant are called initial conditions in network.

1. Resistor:

$$ V = i \times R $$

Equation 1 is linear and also time dependent. This indicates that current through resistor changes if applied voltage changes instantaneously. Thus in resistor, change in current is instantaneous as there is no storage of energy in it.

2. Inductor:

$$ V_L = L \frac{di_L}{dt} $$

If dc current flows through inductor, $\frac{di_L}{dt}$ becomes zero as dc current is constant with respect to time. Hence voltage across inductor, $V_L$ becomes zero. Thus, as far as dc quantities are considered, in steady state, inductor acts as short circuit.

We can express inductor current in terms of voltage developed across it as

$$ i_L = \frac{1}{L} \int_0^t V_L \, dt $$

In above eqn. The limits of integration is from $-\infty$ to $t$.

Assuming that switching takes place at $t=0$, we can split limits into two intervals as $-\infty$ to $0$ and $0$ to $t$.

$$ i_L = \frac{1}{L} \int_{-\infty}^0 V_L \, dt + \frac{1}{L} \int_0^t V_L \, dt $$
\[ i_L = i_L \left( \frac{1}{L} \int V_L \, dt \right) \]

\[ i_L \left( \frac{1}{L} \int V_L \, dt \right) \]

at \( t = 0^+ \) we can write \( i_L(0^+) = i_L(0^-) \)

I through inductor cannot change instantaneously.

**3. Capacitor**

\[ i_C = C \frac{dV_C}{dt} \]

If dc voltage is applied to capacitor, \( \frac{dV_C}{dt} \) becomes zero as dc voltage is constant with respect to time.

Hence the current through capacitor \( i_C \) becomes zero. Thus as far as dc quantities are considered capacitor acts as open circuit.

\[ V_C = \frac{1}{C} \int i_C \, dt \]

\[ V_C = \frac{1}{C} \int_{-\infty}^{t} i_C \, dt \]

Splitting limits of integration

\[ V_C = \frac{1}{C} \int_{-\infty}^{0} i_C \, dt + \frac{1}{C} \int_{0}^{t} i_C \, dt \]

At \( t(0^+) \), equation is given by

\[ V_C \left( \frac{1}{C} \right) \int_{-\infty}^{0} i_c \, dt + \frac{1}{C} \int_{0}^{t} i_c \, dt \]

Thus voltage across capacitor can not change instantaneously.

**4.8. Transient Response of RL Circuits:**

So far we have considered dc resistive network in which currents and voltages were independent of time. More specifically, Voltage (cause input) and current (effect output) responses displayed simultaneously except for a constant multiplicative factor (VR). Two basic passive elements namely, inductor and capacitor are introduced in the dc network. Automatically, the question will arise whether or not the methods developed in lesson-3 to lesson-8 for resistive circuit analysis are still valid. The voltage/current relationship for these two passive elements are defined by the derivative (voltage across the inductor).
where \( i(t) \) is the current flowing through the inductor; current through the capacitor

\[
i_c(t) = C \frac{dv_c(t)}{dt},
\]

voltage across the capacitor or in integral form as \((t)\)

\[
i_L(t) = \frac{1}{L} \int_0^t v_L(t) \, dt + i_L(0) \quad \text{or} \quad v_c(t) = \frac{1}{C} \int_0^t i(t) \, dt + v_c(0)
\]

Our problem is to study the growth of current in the circuit through two stages, namely: (i) dc transient response (ii) steady state response of the system.

D.C Transients: The behavior of the current and the voltage in the circuit switch is closed until it reaches its final value is called dc transient response of the concerned circuit. The response of a circuit (containing resistances, inductances, capacitors and switches) due to sudden application of voltage or current is called transient response. The most common instance of a transient response in a circuit occurs when a switch is turned on or off – a rather common event in an electric circuit.

**Growth or Rise of current in R-L circuit**

To find the current expression (response) for the circuit shown in fig. 10.6(a), we can write the KVL equation around the circuit.

The table shows how the current \( i(t) \) builds up in a R-L circuit.
<table>
<thead>
<tr>
<th>Actual time (t) in sec</th>
<th>Growth of current in inductor (Eq. 10.15)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t = 0 )</td>
<td>( i(0) = 0 )</td>
</tr>
<tr>
<td>( t = \tau \left( = \frac{L}{R} \right) )</td>
<td>( i(\tau) = 0.632 \times \frac{V_i}{R} )</td>
</tr>
<tr>
<td>( t = 2\tau )</td>
<td>( i(2\tau) = 0.865 \times \frac{V_i}{R} )</td>
</tr>
<tr>
<td>( t = 3\tau )</td>
<td>( i(3\tau) = 0.950 \times \frac{V_i}{R} )</td>
</tr>
<tr>
<td>( t = 4\tau )</td>
<td>( i(4\tau) = 0.982 \times \frac{V_i}{R} )</td>
</tr>
<tr>
<td>( t = 5\tau )</td>
<td>( i(5\tau) = 0.993 \times \frac{V_i}{R} )</td>
</tr>
</tbody>
</table>

![Graph showing the growth of current in an inductor over time]
Consider network shown in fig. the switch \( k \) is moved from position 1 to 2 at reference time \( t = 0 \).

Now before switching take place, the capacitor \( C \) is fully charged to \( V \) volts and it discharges through resistance \( R \). As time passes, charge and hence voltage across capacitor i.e. \( V_c \) decreases gradually and hence discharge current also decreases gradually from maximum to zero exponentially.

After switching has taken place, applying kirchoff's voltage law,

\[
0 = V_R + V_C
\]

Where \( V_R \) is voltage across resistor and \( V_C \) is voltage across capacitor.

\[
\therefore V_c = -V_R = -i \cdot R
\]

\[
i = C \frac{dV_c}{dt}
\]

\[
\therefore V_c = -R \cdot C \cdot \frac{dV_c}{dt}
\]

Above equation is linear, homogenous first order differential equation. Hence rearranging we have,

\[
\frac{dt}{RC} = -\frac{dV_c}{V_c}
\]

Integrating both sides of above equation we have
\[ \frac{t}{RC} = -\ln V_C + k' \]

Now at \( t = 0 \), \( V_C = V \) which is initial condition, substituting in equation we have,

\[ \therefore 0 = -\ln V + k' \]

\[ \therefore k' = \ln V \]

Substituting value of \( k' \) in general solution, we have

\[ \frac{t}{RC} = -\ln V_C + \ln V \]

\[ \therefore \frac{t}{RC} = \ln \frac{V}{V_C} \]

\[ \therefore \frac{V}{V_C} = e^{\frac{t}{RC}} \]

\[ \therefore V_C = V \cdot e^{-\frac{t}{RC}} \]

\[ V = \frac{Q}{C} \]

Where \( Q \) is total charge on capacitor

Similarly at any instant, \( V_C = q/c \) where \( q \) is instantaneous charge.

So we have,

\[ \frac{q}{c} = \frac{Q}{C} e^{-\frac{t}{RC}} \]

\[ q = Q \cdot e^{-\frac{t}{RC}} \]

Thus charge behaves similarly to voltage across capacitor.

Now discharging current \( i \) is given by

\[ i = \frac{V_C}{R} \]

but \( V_R = V_C \) when there is no source in circuit.

\[ \therefore i = \frac{V_C}{R} \]

\[ \therefore i = \frac{V}{R} e^{-\frac{t}{RC}} \]

The above expression is nothing but discharge current of capacitor. The variation of this current with respect to time is shown in fig.

This shows that the current is exponentially decaying. At point P on the graph. The current value is (0.368) times its maximum value. The characteristics of decay are determined by values \( R \) and \( C \), which are 2 parameters of network.

For this network, after the instant \( t = 0 \), there is no driving voltage source in circuit, hence it is called undriven RC circuit.
4.9. TRANSIENT RESPONSE OF RC CIRCUITS

Ideal and real capacitors: An ideal capacitor has an infinite dielectric resistance and plates (made of metals) that have zero resistance. However, an ideal capacitor does not exist as all dielectrics have some leakage current and all capacitor plates have some resistance. A capacitor’s leakage resistance is a measure of how much charge (current) it will allow to leak through the dielectric medium. Ideally, a charged capacitor is not supposed to allow leaking any current through the dielectric medium and also assumed not to dissipate any power loss in capacitor plates resistance. Under this situation, the model as shown in fig. 10.16(a) represents the ideal capacitor. However, all real or practical capacitor leaks current to some extend due to leakage resistance of dielectric medium. This leakage resistance can be visualized as a resistance connected in parallel with the capacitor and power loss in capacitor plates can be realized with a resistance connected in series with capacitor. The model of a real capacitor is shown in fig.

Let us consider a simple series RC-circuit shown in fig. 10.17(a) is connected through a switch ‘S’ to a constant voltage source.

The switch ‘S’ is closed at time ‘t=0’. It is assumed that the capacitor is initially charged with a voltage and the current flowing through the circuit at any instant of time ‘t’ after closing the switch is

**Current decay in source free series RL circuit:**

At t = 0’, switch k is kept at position ‘a’ for very long time. Thus, the network is in steady state. Initial current through inductor is given as,

\[ i_L = I_0 = \frac{V}{R} = i_L \]

Because current through inductor can not change instantaneously
Assume that at t = 0 switch k is moved to position ‘b’,
Applying KVL,
\[ L \frac{di}{dt} + iR = 0 \]
\[ \Rightarrow L \frac{di}{dt} = -iR \]

Rearranging the terms in above equation by separating variables
\[ \frac{di}{i} = \frac{-R}{L} dt \]

Integrating both sides with respect to corresponding variables
\[ \Rightarrow \ln \left( \frac{L}{R} \right) = \frac{-R}{L} \cdot t + k' \]
\[ k' = \ln \left( \frac{L}{R} \right) \]

Where \( k' \) is constant of integration.

To find \( k' \):

Form equation 1, at \( t=0, i=I_0 \)
Substituting the values in equation 3
\[ \Rightarrow \ln \left( \frac{L}{R} \right) = \frac{-R}{L} \cdot 0 + k' \]
\[ k' = \ln \left( \frac{L}{R} \right) \]

Substituting value of \( k' \) from equation 4 in equation 3
\[ \ln \left( \frac{L}{R} \right) = \frac{-R}{L} \cdot t + \ln \left( \frac{L}{R} \right) \]
\[ \Rightarrow \frac{L}{R} \cdot \ln \left( \frac{L}{R} \right) = \frac{-R}{L} \cdot t \]
\[ \Rightarrow \frac{i}{I_0} = e^{\frac{R}{L} \cdot t} \]
\[ \Rightarrow i = I_0 \cdot e^{\frac{R}{L} \cdot t} \]

fig. shows variation of current \( i \) with respect to time

From the graph, H is clear that current is exponentially decaying. At point P on graph. The current value is \((0.363)\) times its maximum value. The characteristics of decay are determined by values \( R \) and \( L \) which are two parameters of network.

The voltage across inductor is given by
\[ V_L = L \frac{di}{dt} = L \frac{d}{dt} \left[ I_0 \cdot e^{\frac{R}{L} \cdot t} \right] = L \cdot I_0 \left( -\frac{R}{L} \right) \cdot e^{\frac{R}{L} \cdot t} \]
\[
V_L = -I_0 \cdot R \cdot L
\]

\[
\therefore V_L = -V \cdot e^{-\frac{R}{L}t}
\]

4.10. **TRANSIENT RESPONSE OF RLC CIRCUITS**

In the preceding lesson, our discussion focused extensively on dc circuits having resistances with either inductor (\(L\)) or capacitor (\(C\)) (i.e., single storage element) but not both. Dynamic response of such first order system has been studied and discussed in detail. The presence of resistance, inductance, and capacitance in the dc circuit introduces at least a second order differential equation or by two simultaneous coupled linear first order differential equations. We shall see in next section that the complexity of analysis of second order circuits increases significantly when compared with that encountered with first order circuits. Initial conditions for the circuit variables and their derivatives play an important role and this is very crucial to analyze a second order dynamic system.

**Response of a series R-L-C circuit**

Consider a series RL circuit as shown in fig. 11.1, and it is excited with a dc voltage source \(C \rightarrow sV\). Applying around the closed path for,

\[
L \frac{di(t)}{dt} + Ri(t) + v_c(t) = V_s
\]

![Fig. 11.1: A Simple R-L-C circuit excited with a dc voltage source](image)

The current through the capacitor can be written as

\[
i(t) = C \frac{dv_c(t)}{dt}
\]

\[
LC \frac{d^2v_c(t)}{dt^2} + RC \frac{dv_c(t)}{dt} + v_c(t) = V_s
\]

The above equation is a 2nd-order linear differential equation and the parameters associated with the differential equation are constant with time. The complete solution of the above differential equation has two components; the transient response and the steady state response. Mathematically, one can write the complete solution as

\[
v_c(t) = v_{cn}(t) + v_{cs}(t) = \left( A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t} \right) + A
\]
Since the system is linear, the nature of steady state response is same as that of forcing function (input voltage) and it is given by a constant value. Now, the first part of the total response is completely dies out with time while and it is defined as a transient or natural response of the system. The natural or transient response (see Appendix in Lesson-10) of second order differential equation can be obtained from the homogeneous equation (i.e., from force free system) that is expressed by

$$a \frac{d^2v_e(t)}{dt^2} + b \frac{dv_e(t)}{dt} + c v_e(t) = 0 \quad \text{(where \ a = 1, \ b = \frac{R}{L} \ and \ c = \frac{1}{LC})}$$

and solving the roots of this equation (11.5) one can find the constants $\lambda$ and $\alpha$ of the exponential terms that associated with transient part of the complete solution (eq.11.3) and they are given below.

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

The roots of the characteristic equation are classified in three groups depending upon the values of the parameters $R, L, C$ and of the circuit

**Case-A (overdamped response):** That the roots are distinct with negative real parts. Under this situation, the natural or transient part of the complete solution is written as

$$v_{cn}(t) = A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t}$$

and each term of the above expression decays exponentially and ultimately reduces to zero as and it is termed as overdamped response of input free system. A system that is overdamped responds slowly to any change in excitation. It may be noted that the exponential term \( t \to \infty \) \( A_1 e^{\alpha_1 t} \) takes longer time to decay its value to zero than the term \( t \to \infty \) \( A_2 e^{\alpha_2 t} \). One can introduce a factor $\xi$ that provides an information about the speed of system response and it is defined by damping ratio

$$\xi = \frac{\text{Actual damping}}{\text{critical damping}} = \frac{b}{2a} = \frac{R/L}{2\sqrt{ac}} > 1$$

**RLC Circuit:**
Consider a circuit in which R, L, and C are connected in series with each other across ac supply as shown in fig. The ac supply is given by,

\[ V = V_m \sin wt \]

The circuit draws a current I. Due to that different voltage drops are,

1. Voltage drop across Resistance R is \( V_R = IR \)
2. Voltage drop across Inductance L is \( V_L = IX_L \)
3. Voltage drop across Capacitance C is \( V_C = IX_C \)

The characteristics of three drops are,

(i) \( V_R \) is in phase with current I
(ii) \( V_L \) leads I by 90°
(iii) \( V_C \) lags I by 90°

According to krichoff’s laws

**Steps to draw phasor diagram:**

1. Take current I as reference
2. \( V_R \) is in phase with current I
3. \( V_L \) leads current by 90°
4. \( V_C \) lags current by 90°
5. obtain resultant of \( V_L \) and \( V_C \). Both \( V_L \) and \( V_C \) are in phase opposition (180° out of phase)
6. Add that with \( V_R \) by law of parallelogram to get supply voltage.

The phasor diagram depends on the condition of magnitude of \( V_L \) and \( V_C \) which ultimately depends on values of \( X_L \) and \( X_C \).

Let us consider different cases:

**Case(i): \( X_L > X_C \)**

When \( X_L > X_C \)

Also \( V_L > V_C \) (or) \( IX_L > IX_C \)

So, resultant of \( V_L \) and \( V_C \) will directed towards \( V_L \) i.e. leading current I. Hence I lags V i.e. current I will lags the resultant of \( V_L \) and \( V_C \) i.e. \((V_L - V_C)\). The circuit is said to be inductive in nature.

From voltage triangle,

\[ V = \sqrt{(V_R^2 + (V_L - V_C)^2)} = \sqrt{((IR)^2 + (IX_L - IX_C)^2)} \]
\[ V = I \sqrt{(R^2 + (X_L - X_C)^2)} \]
\[ V = IZ \]
\[ Z = \sqrt{(R^2 + (X_L - X_C)^2)} \]
If \( V = V_m \sin \omega t \); \( i = I_m \sin (\omega t - \phi) \)
i.e I lags V by angle \( \phi \)

**Case(ii): \( X_L < X_c \)**

When \( X_L < X_c \)
Also \( V_L < V_c \) (or) \( IX_L < IX_c \)
Hence the resultant of \( V_L \) and \( V_c \) will directed towards \( V_c \) i.e current is said to be capacitive in nature

Form voltage triangle
\[
V = \sqrt{(V_R^2 + (V_c - V_L)^2)} = \sqrt{(IR)^2 + (IX_c - IX_L)^2)}
\]
\[
V = IZ
\]
\[
Z = \sqrt{(R^2 + (X_c - X_L)^2)}
\]

If \( V = V_m \sin \omega t \); \( i = I_m \sin (\omega t + \phi) \)
i.e I lags V by angle \( \phi \)

**Case(iii): \( X_L = X_c \)**

When \( X_L = X_c \)
Also \( V_L = V_c \) (or) \( IX_L = IX_c \)
So \( V_L \) and \( V_c \) cancel each other and the resultant is zero. So \( V = V_R \) in such a case, the circuit is purely resistive in nature.

**Impedance:**

In general for RLC series circuit impedance is given by,
\[
Z = R + j X
\]
\[
X = X_L - X_c = \text{Total reactance of the circuit}
\]
If \( X_L > X_c \); \( X \) is positive & circuit is Inductive
If \( X_L < X_c \); \( X \) is negative & circuit is Capacitive
If \( X_L = X_c \); \( X = 0 \) & circuit is purely Resistive
\[
\tan \phi = \frac{(X_L - X_c)R}{R^2 + (X_L - X_c)^2}
\]
\[
\cos \phi = \frac{R}{Z}
\]
\[
Z = \sqrt{(R^2 + (X_L - X_c)^2)}
\]

**Impedance triangle:**

In both cases
\[
R = Z \cos \phi
\]
\[
X = Z \sin \phi
\]

**Power and power triangle:**

The average power consumed by circuit is,
\[
P_{avg} = (\text{Average power consumed by R}) + (\text{Average power consumed by L}) + (\text{Average power consumed by C})
\]
\[
P_{avg} = \text{Power taken by R} = I^2R
\]
\[
= I(\text{R}) = VI
\]
\[
V = V \cos \phi
\]
\[
P = VI \cos \phi
\]
Thus, for any condition, \( X_L > X_c \) or \( X_L < X_c \)
General power can be expressed as
\[
P = \text{Voltage} \times \text{Component in phase with voltage}
\]

**Power triangle:**

\[
S = \text{Apparent power} = I^2Z = VI
\]
\[
P = \text{Real or True power} = VI \cos \phi = \text{Active power}
\]
\[
Q = \text{Reactive power} = VI \sin \phi
\]

---

4.11. **CHARACTERIZATION OF TWO PORT NETWORKS IN TERMS OF Z,Y AND H PARAMETERS.**

The purpose of this appendix is to study methods of characterizing and analyzing two-port networks. A port is a terminal pair where energy can be supplied or extracted. A two-port network is
a four-terminal circuit in which the terminals are paired to form an input port and an output port. Figure W2–1 shows the customary way of defining the port voltages and currents. Note that the reference marks for the port variables comply with the passive sign convention.

The linear circuit connecting the two ports is assumed to be in the zero state and to be free of any independent sources. In other words, there is no initial energy stored in the circuit and the box in Figure W2–1 contains only resistors, capacitors, inductors, mutual inductance, and dependent sources. A four-terminal network qualifies as a two-port if the net current entering each terminal pair is zero. This means that the current exiting the lower port terminals in Figure W2–1 must be equal to the currents entering the upper terminals.

One way to meet this condition is to always connect external sources and loads between the input terminal pair or between the output terminal pair. The first task is to identify circuit parameters that characterize a two-port. In the two port approach the only available variables are the port voltages $V_1$ and $V_2$, and the port currents $I_1$ and $I_2$. A set of two-port parameters is defined by expressing two of these four-port variables in terms of the other two variables. In this appendix we study the four ways in Table W2–1.

### TWO-PORT PARAMETERS

<table>
<thead>
<tr>
<th>Name</th>
<th>Express</th>
<th>In Terms Of</th>
<th>Defining Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impedance</td>
<td>$V_1, V_2$</td>
<td>$I_1, I_2$</td>
<td>$V_1 = z_{11}I_1 + z_{12}I_2$ and $V_2 = z_{21}I_1 + z_{22}I_2$</td>
</tr>
<tr>
<td>Admittance</td>
<td>$I_1, I_2$</td>
<td>$V_1, V_2$</td>
<td>$I_1 = y_{11}V_1 + y_{12}V_2$ and $I_2 = y_{21}V_1 + y_{22}V_2$</td>
</tr>
<tr>
<td>Hybrid</td>
<td>$V_1, I_1$</td>
<td>$I_1, V_2$</td>
<td>$V_1 = h_{11}I_1 + h_{12}V_2$ and $I_2 = h_{21}I_1 + h_{22}V_2$</td>
</tr>
<tr>
<td>Transmission</td>
<td>$V_1, I_1$</td>
<td>$V_2, -I_2$</td>
<td>$V_1 = A V_2 - B I_2$ and $I_1 = C V_2 - D I_2$</td>
</tr>
</tbody>
</table>

Note that each set of parameters is defined by two equations, one for each of the two dependent port variables. Each equation involves a sum of two terms, one for each of the two independent port variables. Each term involves a proportionality because the two-port is a linear circuit and superposition applies. The names given the parameters indicate their dimensions (impedance and admittance), a mixture of dimensions (hybrid), or their original application (transmission lines). With double-subscripted parameters, the first subscript indicates the port at which the dependent variable appears and the second subscript the port at which the independent variable appears.

Regardless of their dimensions, all two-port parameters are network functions. In general, the parameters are functions of the complex frequency variable and s-domain circuit analysis applies. For sinusoidal steady-state problems, we replace $s$ by $j\omega$ and use phasor circuit analysis. For purely resistive circuits, the two-port parameters are real constants and we use resistive circuit analysis. Before turning to specific parameters, it is important to specify the objectives of two-port network analysis. Briefly, these objectives are:

1. Determine two-port parameters of a given circuit.
2. Use two-port parameters to find port variable responses for specified input sources and output loads.
In principle, the port variable responses can be found by applying node or mesh analysis to the internal circuitry connecting the input and output ports. So why adopt the two-port point of view? Why not use straightforward circuit analysis?

There are several reasons. First, two-port parameters can be determined experimentally without resorting to circuit analysis. Second, there are applications in power systems and microwave circuits in which input and output ports are the only places that signals can be measured or observed. Finally, once two-port parameters of a circuit are known, it is relatively simple to find port variable responses for different input sources and/or different output loads.

Two ports are the only places that signals can be measured or observed. Finally, once two-port parameters of a circuit are known, it is relatively simple to find port variable responses for different input sources and/or different output loads.

**IMPEDANCE PARAMETERS**

The impedance parameters are obtained by expressing the port voltages $V_1$ and $V_2$ in terms of the port currents $I_1$ and $I_2$.

\[
V_1 = z_{11}I_1 + z_{12}I_2
\]

\[
V_2 = z_{21}I_1 + z_{22}I_2
\]  

(W2–1)

The network functions $z_{11}$, $z_{12}$, $z_{21}$, and $z_{22}$ are called the impedance parameters or simply the $z$-parameters. The matrix form of these equations are

\[
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} = \begin{bmatrix}
z_{11} & z_{12} \\
z_{21} & z_{22}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix} = [z] \begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
\]  

(W2–2)

where the matrix $[z]$ is called the impedance matrix of a two-port network. To measure or compute the impedance parameters, we apply excitation at one port and leave the other port open-circuited. When we drive port 1 with port 2 open ($I_2 = 0$), the expressions in Eq. (W2–1) reduce to one term each, and yield the definitions of $z_{11}$ and $z_{21}$.

\[
z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2 = 0} = \text{input impedance with the output port open} \tag{W2–3a}
\]

\[
z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2 = 0} = \text{forward transfer impedance with the output port open}
\]

Conversely, when we drive port 2 with port 1 open ($I_1 = 0$), the expressions in Eq. (W2–1) reduce to one term each that define $z_{12}$ and $z_{22}$ as

\[
z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1 = 0} = \text{reverse transfer impedance with the input port open} \tag{W2–4a}
\]

\[
z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1 = 0} = \text{output impedance with the input port open} \tag{W2–4b}
\]

All of these parameters are impedances with dimensions of ohms. A two-port is said to be reciprocal when the open-circuit voltage measured at one port due to a current excitation at the other port is unchanged when the measurement and excitation ports are interchanged. A two-port that fails this test is said to be nonreciprocal. Circuits containing resistors, capacitors, and inductors (including mutual inductance) are always reciprocal. Adding dependent sources to the mix usually makes the two-port nonreciprocal. If a two-port is reciprocal, then $z_{12} = z_{21}$. To prove this we apply an excitation $I_1$ to the input port and observe that Eq. (W2–1) gives the open-circuit ($I_2 = 0$) voltage at the output port as $V_{20C} = z_{21}I_1$. Reversing the excitation and observation ports, we find that an excitation $I_2$ produces an open-circuit ($I_1 = 0$) voltage at the input port of $V_{10C} = z_{12}I_2$. Reciprocity requires that $V_{10C} = V_{20C}$, which can only happen if $z_{12} = z_{21}$.

**ADMITTANCE PARAMETERS**
The admittance parameters are obtained by expressing the port currents $I_1$ and $I_2$ in terms of the port voltages $V_1$ and $V_2$. The resulting two-port i–v relationships are

$$I_1 = y_{11} V_1 + y_{12} V_2$$
$$I_2 = y_{21} V_1 + y_{22} V_2$$ (W2–5)

The network functions $y_{11}$, $y_{12}$, $y_{21}$, and $y_{22}$ are called the admittance parameters or simply the $y$-parameters. In matrix form these equations are

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$ (W2–6)

where the matrix $[y]$ is called the admittance matrix of a two-port network. To measure or compute the admittance parameters, we apply excitation at one port and short circuit the other port. When we drive at port 1 with port 2 shorted ($V_2 = 0$), the expressions in Eq. (W2–5) reduce to one term each that define $y_{11}$ and $y_{21}$ as

$$y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0} = \text{input admittance with the output port shorted} \quad \text{(W2–7a)}$$

$$y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0} = \text{forward transfer admittance with the output port shorted}$$

Conversely, when we drive at port 2 with port 1 shorted ($V_1 = 0$), the expressions in Eq. (W2–5) reduce to one term each that define $y_{22}$ and $y_{12}$ as

$$y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1=0} = \text{reverse transfer admittance with the input port shorted} \quad \text{(W2–8a)}$$

$$y_{22} = \left. \frac{I_2}{V_2} \right|_{V_1=0} = \text{output admittance with the input port shorted} \quad \text{(W2–8b)}$$

All of these network functions are admittances with dimensions of Siemens. If a two-port is reciprocal, then $y_{12} = y_{21}$. This can be proved using the same process applied to the $z$-parameters. The admittance parameters express port currents in terms of port voltages, whereas the impedance parameters express the port voltages in terms of the port currents. In effect these parameters are inverses. To see this mathematically, we multiply Eq. (W2–2) by $[z]^{-1}$, the inverse of the impedance matrix.

$$[z]^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [z]^{-1}[z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

In other words,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [z]^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

HYBRID PARAMETERS

The hybrid parameters are defined in terms of a mixture of port variables. Specifically, these parameters express $V_1$ and $I_2$ in terms of $I_1$ and $V_2$. The resulting two-port i–v relationships are

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$ (W2–9)

Where $h_{11}$, $h_{12}$, $h_{21}$, and $h_{22}$ are called the hybrid parameters or simply the h-parameters. In matrix form these equations are
Where the matrix \([h]\) is called the h-matrix of a two-port network. The h-parameters can be measured or calculated as follows. When we drive at port 1 with port 2 shorted \((V_2=0)\), the expressions in Eq. (W2–9) reduce to one term each, and yield the definitions of \(h_{11}\) and \(h_{21}\).

\[
\begin{bmatrix}
V_1 \\
I_2
\end{bmatrix} = \begin{bmatrix}
h_{11} & h_{12} \\
h_{21} & h_{22}
\end{bmatrix} \begin{bmatrix}
I_1 \\
V_2
\end{bmatrix} = [h] \begin{bmatrix}
I_1 \\
V_2
\end{bmatrix} \tag{W2–10}
\]

\[
h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} = \text{input impedance with the output port shorted} \tag{W2–11a}
\]

\[
h_{21} = \left. \frac{I_2}{I_1} \right|_{V_2=0} = \text{forward current transfer function with the output port shorted} \tag{W2–11b}
\]

When we drive at port 2 with port 1 open \((I_1=0)\), the expressions in Eq. (W2–9) reduce to one term each, and yield the definitions of \(h_{12}\) and \(h_{22}\).

\[
h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0} = \text{reverse voltage transfer function with the input port open} \tag{W2–12a}
\]

\[
h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0} = \text{output admittance with the input port open} \tag{W2–12b}
\]

These network functions have a mixture of dimensions: \(h_{11}\) is impedance in ohms, \(h_{22}\) is admittance in Siemens, and \(h_{21}\) and \(h_{12}\) are dimensionless transfer functions. If a two-port is reciprocal, then \(h_{12}h_{21}\). This can be proved by the same method applied to the z-parameters.
1. Define response.
The current flowing through or voltage across branches in the circuit is called response.

2. Define steady state response.
The behavior of voltage or current does not change with time is called steady state response.

3. Define transient response.
The voltage or current are changed from one transient state to another transient state is called transient response.

4. Define natural response. (MAY-08)
The response determined by the internal energy stored in the network is called natural response. It depends upon the type of elements, their size and the interconnection of elements. The response is independent of the source.

   Energy may be stored internally in the form of electric field of capacitor or in the magnetic field of an inductor.

   The natural response also known as transient response.

5. Define forced response.
The response determined by the application of external energy source is called forced response. The external energy source of forcing function may be direct voltage or current, sinusoidal source, exponential function, ramp function.

6. What is transient? [NOV/DEC’07]
The state (or condition) of the circuit from the transient of switching to attainment of steady state is called transient state or simply transient.

7. Why transient occurs in electric circuits?
The inductance will not allow the sudden change in current and the capacitance will not allow sudden change in voltage. Hence inductive and capacitive circuits (or in general reactive circuits) transient occurs during switching operation.

8. Define time constant of RL circuit. [NOV/DEC’07] (JUN-09) (MAY-08) [DEC-10]
   \[ \tau = \frac{L}{R} \]
   The time constant of RL circuit is defined as the ratio of inductance and resistance of the circuit.

   (Or)
   The time constant of RL circuit is defined as the time taken by the current through the inductance to reach 63.21% of its final steady state value.

   (Or)
   The time constant of RL circuit is defined as the time taken by the voltage across inductance to fall to 36.79% of its initial value.

   (Or)
   The time constant of RL circuit is defined as the time taken by current through the inductance to reach steady state value if initial rate of rise is maintained.

9. Define time constant of RC circuit. [MAY/JUNE’07] (JUN-09) (MAY-08) [DEC-10]
   \[ \tau = RC \]
   The time constant of RC circuit is defined as the product of capacitance and resistance of the circuit.

   (Or)
   The time constant of RC circuit is defined as the time taken by the voltage across the capacitance to reach 63.21% of its final steady state value.

   (Or)
   The time constant of RC circuit is defined as the time taken by the current through the capacitance to fall to 36.79% of its initial value.
(Or)

The time constant of RL circuit is defined as the time taken by the voltage across the capacitance to reach steady state value if initial rate of rise is maintained.

10. Sketch the transient current and voltages of RL circuit.

11. Sketch the transient current and voltages of RC circuit.


\[ i_L = C \frac{dV_C}{dt} \]

If the voltage across capacitance changes instantaneously then the current \( I_L = \alpha \) which is impossible since it requires infinite power. Hence voltage across capacitor cannot change instantaneously.


\[ V_i = L \frac{di_L}{dt} \]

If the current through inductor changes instantaneously then the current \( V_L = \alpha \) which is impossible since it requires infinite power. Hence current through inductor cannot change instantaneously.

14. What is the initial condition of the elements capacitor and inductor that have no initial energy storage?

The capacitor acts as a short circuit and the inductor acts as an open circuit.

15. What is the final condition of the elements inductor and capacitor?

The capacitor acts as an open circuit and the inductor acts as a short circuit.

16. Write the conditions for response of an RLC series network?

Under damped response:

\[ \frac{R^2}{4L^2} < \frac{1}{LC} \]

Over damped response:

\[ \frac{R^2}{4L^2} > \frac{1}{LC} \]

Critically damped response:

\[ \frac{R^2}{4L^2} = \frac{1}{LC} \]

17. Sketch the response of RLC circuit.

18. What is the time constant of RL circuit with \( R=10\Omega \) and \( L=20\text{mH} \).

Time constant \( \tau = \frac{L}{R} = \frac{20 \times 10^{-3}}{10} = 2\text{ms} \)
19. What is the time constant of RL circuit.

\[ \tau = \frac{L}{R} = \frac{0.1429}{12} = 0.0119 \text{s} \]

20. What is the time constant of RC circuit with \(R=10\,\text{KΩ}\) and \(C=40\,\mu\text{F}\).

\[ \tau = RC = 10 \times 10^3 \times 40 \times 10^{-6} = 0.4 \text{s} \]

21. What is the time constant of RC circuit.

\[ \tau = RC = 8 \times 0.5 = 4 \text{s} \]

22. What is damping ratio?

The ratio of resistance of the circuit and resistance for critical damping is called damping ratio.

23. What is critical damping?

The critical damping is the condition of the circuit at which the oscillations in the response are just eliminated. This is possible by increasing the value of resistance in the circuit.

24. What is critical resistance?

The critical resistance is the value of the resistance of the circuit to achieve critical damping.

25. Write the expression for critical resistance and damping ratio of RLC series circuit. (JUN-09)

\[ \text{Critical resistance } R_c = 2 \sqrt{\frac{L}{C}} \]

\[ \text{Damping ratio } \xi = \frac{R}{2}\sqrt{\frac{C}{L}} \]

26. What is natural and damped frequency?

The response of the circuit is completely oscillatory with a frequency \(\omega_n\) in the absence of resistance and this frequency \(\omega_n\) is called natural frequency.

The response of under damped circuit is oscillatory with a frequency of \(\omega_n\) and these oscillations are damped as \(t\) tends to infinity. The frequency of damped oscillatory response is called damped frequency.

27. A RC series circuit is excited by a dc voltage source 80V by closing the switch at \(t=0\). Determine the voltage across the capacitor in a time of one constant.

\[ V_{C(t)} = V \left(1 - e^{-\frac{t}{\tau}}\right) = 80 \left(1 - e^{-\frac{1}{0.0119}}\right) = 50.5696 \text{V} \]

28. A RL series circuit with \(R=10\,\Omega\) is excited by a dc voltage source 30V by closing the switch at \(t=0\). Determine the current in the circuit at \(t=2\,\tau\)

\[ I_{\text{through the inductor}} = \frac{V}{R} \left(1 - e^{-\frac{t}{\tau}}\right) = \frac{30}{10} \left(1 - e^{-\frac{2}{0.0119}}\right) = 2.594 \text{A} \]

29. A RLC series circuit with \(R=5\,\Omega\) is excited by a dc voltage source 10V by closing the switch at \(t=0\). Draw the initial and final condition of the circuit.

30. Sketch the transient current given by \(i = 5 - 4e^{-20t}\)

\[ i = \begin{cases} 5 - 4e^{-20t} & \text{for } t > 0 \\ 5 & \text{for } t = 0 \end{cases} \]

\[ i = \begin{cases} 4 & \text{for } t > 0 \\ 2.5 - e^{-20t} & \text{for } t < 0 \end{cases} \]
31. A pure inductance of 1.273 mH is connected in series with a pure resistance of 30 Ω. If the frequency of the sinusoidal supply is 5 kHz and the p.d. across the 30Ω resistor is 6 V, determine the value of the supply voltage and the voltage across the 1.273 mH inductance. Draw the phasor diagram. (MAY-07)

The circuit is shown in Figure
Supply voltage, \( V = IZ \)
Current \( I = \frac{Vr}{R} = \frac{6}{30} = 0.20 \) A
Inductive reactance \( X_L = 2\pi fL = 2\pi (5 \times 10^3)(1.273 \times 10^{-3}) = 40 \) Ω
Impedance, \( Z = \sqrt{R^2 + X^2} = \sqrt{30^2 + 40^2} = 50 \) Ω
Supply voltage \( V = IZ = (0.20)(50) = 10 \) V
Voltage across the 1.273 mH inductance, \( V_L = IX_L = (0.2)(40) = 8 \) V
The phasor diagram is shown in Figure.
(Note that in a.c. circuits, the supply voltage is not the arithmetic sum of the p.d.’s across components but the phasor sum)
PART-B  
16 MARKS

1. Show that
(i) Current through purely resistive circuit is in phase the applied voltage.
(ii) Current through pure inductance lags applied voltage by 90°
(iii) Current through pure capacitance leads applied voltage by 90°

AC through pure resistance:

Consider a simple circuit consisting of a pure resistance ‘R’ ohms across voltage

\[ V = V_m \sin(\omega t) \]

According to ohms law,

\[ i = \frac{V}{R} = (V_m \sin(\omega t)) \frac{1}{R} \]

\[ i = (V_m R) \sin(\omega t) \]

This is equation giving instantaneous value of current

\[ i = I_m \sin(\omega t + \phi) \]

\[ I_m = \frac{V_m}{R} \quad \text{and} \quad \phi = 0 \]

It is in phase with the voltage applied. There is no phase different between two.

“In purely resistive circuit, the current and the voltage applied are in phase with each other “

Power:

The instantaneous power in a.c circuit can be obtained by taking product of the instantaneous value of current and voltage.

\[ P = V x I \]

\[ = V_m \sin(\omega t) \times I_m \sin(\omega t) \]

\[ = V_m I_m \sin^2(\omega t) \]

\[ = (V_m I_m) \times (1 - \cos(2\omega t)) \]

\[ P = (V_m I_m) - (V_m I_m) \cos(2\omega t) \]

Instantaneous power consists of two components:

1- Constant power component \((V_m I_m)\)

2- Fluctuating component \([(V_m I_m) \cos(2\omega t)]\) having frequency, double the frequency of applied voltage.

The average value of fluctuating cosine component of double frequency is zero, over one complete cycle. So, average power consumption over one cycle is equal to constant power component i.e. \(V_m I_m\)
Pavg = VmIm/2 = (Vm/√2) x (Im/√2)

Pavg = Vrms x Irms  watts

Pavg = VxI watts =I²R watt

AC through pure inductance:

Consider a simple circuit consisting of a pure inductance of L henries connected across a voltage given by the equation.

\[ V = V_m \sin(\omega t) \]

Pure inductance has zero ohmic resistance its internal resistance is zero.

The coil has pure inductance of h henries (H).

When alternating current ‘i’ flows through inductance ‘L’. It sets up an alternating magnetic field around the inductance. This changing flux links the coil and due to self inductance emf gets induced in the coil. This emf opposes the applied voltage.

The self induced emf in the coil is given by

Self induced emf \( e = -L \frac{di}{dt} \)

At all instants, applied voltage \( V \) is equal and opposite to self induced emf \( e \)

\[ V = -e = -(-L \frac{di}{dt}) \]

\[ V = L \frac{di}{dt} \]

\[ V_m \sin(\omega t) = L \frac{di}{dt} \]

\[ di = (V_m/L) \sin(\omega t) \, dt \]

\[ i = \int di = \int (V_m/L) \sin(\omega t) \, dt \]

\[ = (V_m/L)[-\cos(\omega t)/\omega] \]

\[ i = -(V_m/wL) \sin((\pi/2)-\omega t) \]

\[ \Rightarrow \cos(\omega t) = \sin(\omega t - \pi/2) \]

\[ \sin((\pi/2)-\omega t) = -\sin(\omega t - \pi/2) \]

\[ i = I_m \sin(\omega t - \pi/2) \]

Where, \( I_m = V_m/wL = V_m X_L \)

\[ X_L = wL = 2\pi fL \, \Omega \]

The above equation clearly shows that the current is purely sinusoidal and having angle of - \( \pi/2 \) radians i.e. 90°. This means current lags voltage applied by 90°

**Concepts of Induction Reactance:**

\[ I_m = V_m X_L \]

Where, \( X_L = wL = 2\pi fL \, \Omega \)

\( X_L = \) Induction Reactance

Inductive reactance is defined as the opposition offered by the inductance of circuit to the flow of an alternating sinusoidal current.

**Note:**
If frequency is zero, which is so for dc voltage, the inductive reactance is zero. Therefore it is said that inductance offers zero reactance for dc or steady current.

**Power:**

\[
P = VxI = V_m \sin(w t) \times I_m \sin(w t - \pi/2)
\]

\[
= -V_m I_m \sin(w t) \cos(w t)
\]

\[
P = (-V_m I_m) \times \sin(2w t)
\]

The average value of Sine curve over a complete cycle is always zero.

\[
P_{avg} = \int_{0}^{2\pi} V_m I_m \sin(2w t) \, dt = 0
\]

**AC through pure capacitance:**

Consider a simple circuit consisting of pure capacitor of \( C \) farads, connected across a voltage given by equation,

\[
V = V_m \sin(w t)
\]

The current \( I \) charge the capacitor \( C \). The instantaneous charge ‘\( q \)’ on the plates of capacitor is given by

\[
q = CV
\]

\[
q = CV_m \sin(w t)
\]

Current

\[
i = \text{rate of flow of charge ‘} q \text{‘}
\]

\[
i = \frac{dq}{dt} = \frac{d(CV_m \sin(w t))}{dt}
\]

\[
i = CV_m \frac{d(\sin(w t))}{dt}
\]

\[
i = V_m \frac{1}{\omega C} \sin(w t + \pi/2)
\]

\[
i = I_m \sin(w t + \pi/2)
\]

Where, \( I_m = V_m / X_c \)

\[
X_c = \frac{1}{\omega C} = \frac{1}{(2\pi f C)} \Omega
\]

The above equation clearly shows that current is purely sinusoidal and having phase angle of \( \pi/2 \) radians +90°. This means current leads voltage applied by 90°. The positive sign indicates leading nature of the current.

**Concepts of reactive capacitance:**

\[
I_m = V_m / X_c \quad \text{And} \quad X_c = 1/\omega C = 1/(2\pi f C) \Omega
\]

\( X_c = \text{Capacitive reactance} \)

Capacitive reactance is defined as the opposition offered by the capacitance of the circuit to flow of an alternating sinusoidal current.

**Power:**

The expression for instantaneous power can be obtained by taking the product of instantaneous voltage and current

\[
P = V_x I = V_m \sin(w t) \times I_m \sin(w t + \pi/2)
\]

\[
= V_m I_m \sin(w t) \cos(w t)
\]

\[
P = (V_m I_m) \sin(2w t)
\]

\[
P_{avg} = P_{avg} = \int_{0}^{2\pi} (V_m I_m) \sin(2w t) \, d(w t) = 0
\]

2. Show that

i) Current lags voltage in R-L series circuit

ii) Current leads voltage in R-C series circuit

**Solution:**

Current lags voltage in R-L series circuit
Consider a circuit consisting of pure Resistance $R$ ohms connected in series with Inductance $L$ henries as shown in fig.

The series combination is connected across ac supply is given by

$V = V_m \sin \omega t$

The voltage drops in the circuit are,

- Drop across pure resistance $V_R = IR$
- Drop across pure inductance $V_L = IX_L$

Where $X_L = 2\pi fL$

$I = \text{rms value of current drawn}

V_R, V_L = \text{rms value of pure inductance}$

By applying KVL,

$V = IR + IX_L$

**Steps to draw phasor diagram:**

1. I as reference phasor
2. R, V, I are in phase, So $V_R$ will be along I phase in case of resistance
3. I lags voltage by $90^\circ$. But I is reference, $V_L$ must be shown leading w.r.t I by $90^\circ$ in case of inductance
4. Supply voltage = Vector sum of 2 vectors $V_L$ and $V_R$ obtained by law of parallelogram

$V = \sqrt{(V_R)^2 + (V_L)^2} = \sqrt{(IR)^2 + (IX_L)^2} = I \sqrt{(R^2 + X_L^2)}$

$V = IZ$

$Z = \sqrt{(R^2 + X_L^2)}$

From voltage triangle we can write,

$\tan \phi = \frac{V_L}{V_R} = \frac{X_L}{R}$

$\cos \phi = \frac{V_R}{V} = \frac{R}{Z}$

$\sin \phi = \frac{V_L}{V} = \frac{X_L}{Z}$

If all the sides of voltage are divided by current, we get triangle called impedance triangle.

Side or triangles are,

1. Resistance R
2. Inductive reactance $X_L$
3. Impedance $Z$

From this impedance triangle

$R = Z \cos \phi$ \hspace{1cm} X component of Impedance = R

$X_L = Z \sin \phi$ \hspace{1cm} Y Component of Impedance = $X_L$

In rectangular form the impedance is denoted as,

$Z = R + jX_L$

While in polar form

$Z = |Z| \angle \phi \Omega$
Where,
\[
|Z| = \sqrt{(R^2 + X_L^2)}
\]
\[
\phi = \tan^{-1}[X_L/R]
\]

**Impedance:**

Impedance is defined as opposition of circuit to flow alternating current. It is denoted by Z and the unit is ohms.

**Power and power triangle:**

The expression for the current in the series R-L circuit is,
\[
i = Im \sin (wt-\phi) \quad \text{as current lags voltage}
\]

Power = \(VI\) 
\[
= Vm \sin (wt) \times Im \sin (wt-\phi)
\]
\[
= VmIm [\sin(wt) \cdot \sin(wt-\phi)]
\]
\[
= VmIm \left[ (\cos \phi - \cos (2wt-\phi))/2 \right]
\]
\[
= (VmIm/2) \times \cos \phi - (VmIm/2) \times \cos (2wt-\phi)
\]

Second term is cosine term whose average value over a cycle is zero.

Average power \(P_{avg}\) = \((VmIm/2) \times \cos \phi \)
\[
= (Vm/\sqrt{2}) \times (Im/\sqrt{2}) \times \cos \phi
\]
\[
P = VI \cos \phi \text{ watt}
\]

The three side of triangle is,
10. VI 
11. VI \cos \phi 
12. VI \sin \phi 

These three terms can be defined as below,

1. **Apparent power** (S):
   It is defined as the product of rms value of voltage (V) and current (I). It is denoted as ‘S’.
   \(S = VI\) unit is VA
   VA – Voltage ampere

2. **Real or true power** (P):
   It is defined as the product of applied voltage and active component of the circuit. It is real component of the apparent power. It is measured in watts (W) or kilowatts (KW)
   \(P = VI \cos \phi \text{ watts}\)

3. **Reactive power** (Q):
   It is defined as product of applied voltage and reactive component of current. It is also defined as the imaginary of apparent power is represented by Q. Unit is VAR
   \(Q = VI \sin \phi \text{ VAR}\)

Where VAR – Volt ampere reactive.

I lead V in R-C series circuit:
Consider a circuit in which resistance $R$ ohms and capacitance $C$ farads is connected across ac supply is given by,

$$ V = V_m \sin \omega t $$

Circuit draws a current $I$, then there are two voltage drops,

5. Drop across pure resistance $V_R = IR$
6. Drop across pure capacitance $V_C = IX_c$

Where, $X_c = \frac{1}{2\pi fC}$

Apply KVL we get,

$$ V = V_R + V_C $$
$$ V = IxR + IxX_C $$

**Steps to draw phasor diagram:**

- Take current as reference phasor.
- In case of resistance, voltage and current are in phase, so, $V_R$ will along current phasor.
- In case of capacitance, current leads voltage $90^0$ i.e. voltage lags by $90^0$. so, $V_C$ is shown downwards (i.e.) lagging current by $90^0$
- The supply voltage being vector sum of these two voltages $V_C$ and $V_R$ obtained by completing parallelogram

Form voltage triangle,

$$ V = \sqrt{\left(V_R^2 + V_C^2\right)} = \sqrt{(I_R^2 + I^2X_c^2)} $$

$$ V = IZ $$

$Z = \sqrt(R^2 + X_c^2)$ is the impedance of circuit

**Impedance:**

Impedance is nothing but opposition of flow of alternating current. It is measured in ohms given by,

$$ Z = \sqrt(R^2 + X_c^2) $$

Where, $X_c = \frac{1}{\sqrt{2\pi fC}} \Omega$ called capacitive reactance.

From voltage triangle if all sides of voltage triangle are divided by current, we get impedance triangle.

The sides of triangle are $R$, $X_c$, $Z$

The $X$ component is $R = Z \cos \phi$

The $Y$ component is $X_c = Z \sin \phi$

But direction of $X_c$ is $-V_e$ direction

$$ Z = R - j X_c \Omega \quad \text{- Rectangular form} $$

$$ Z = |Z| \angle -\phi \Omega \quad \text{- Polar form} $$

Where,

$$ |Z| = \sqrt(R^2 + X_c^2) $$

$$ \phi = \tan^{-1}\frac{-X_c}{R} $$

**Power and power triangle:**

The $I$ leads the $V$ by angle $\phi$ hence,

$$ i = \text{Im} \sin(\omega t + \phi) $$

$$ \text{Power} = V \times i $$

$$ = V_m \sin(\omega t) \times \text{Im} \sin(\omega t + \phi) $$

$$ = V_m \text{Im} [\sin(\omega t) \sin(\omega t + \phi)] $$

$$ = V_m \text{Im} \left[ \cos(-\phi) - \cos(2\omega t + \phi) \right] $$

$$ = (V_m \text{Im}) \cos \phi - (V_m \text{Im}) \cos(2\omega t + \phi) $$

Second term is cosine term whose average value over a cycle is zero.
Average power \( P_{\text{avg}} = \frac{V_m I_m}{2} \times \cos \phi \)

\[ = \left( \frac{V_m}{\sqrt{2}} \right) \times \left( \frac{I_m}{\sqrt{2}} \right) \times \cos \phi \]

\[ P = VI \cos \phi \text{ watts} \]

If we multiply voltage equation by current \( I \) we get power equation.

Thus various powers are,

1. Apparent power (\( S \))
2. Real or true power (\( P \))
3. Reactive power (\( Q \))

Note:

\[ Z = R + jXL = Z = |Z| \angle \phi \Omega \quad ; \quad \phi \text{ is } +\text{Ve for Inductive } Z \]

\[ P = VI \cos \phi \quad ; \quad \cos \phi \text{ is lagging for Inductive circuit} \]

\[ Z = R - jXL = Z = |Z| \angle -\phi \Omega \quad ; \quad \phi \text{ is } -\text{Ve for Capacitive } Z \]

\[ P = VI \cos \phi \quad ; \quad \cos \phi \text{ is leading for Capacitive circuit} \]

3. Draw the phasor diagram for a series RLC circuit energized by a sinusoidal voltage showing the relative position of current, component voltage and applied voltage for the following case:

   a) When \( X_L > X_C \)
   b) When \( X_L < X_C \)
   c) When \( X_L = X_C \). [AU-JUN-12]

### RLC Circuit:

Consider a circuit in which \( R, L, \) and \( C \) are connected in series with each other across ac supply as shown in fig.

The ac supply is given by,

\[ V = V_m \sin \omega t \]

The circuit draws a current \( I \). Due to that different voltage drops are,

1) Voltage drop across Resistance \( R \) is \( V_R = IR \)
2) Voltage drop across Inductance \( L \) is \( V_L = IX_L \)
3) Voltage drop across Capacitance C is \( V_c = IX_c \)

The characteristics of three drops are,
1. \( V_R \) is in phase with current I
2. \( V_L \) leads I by 90°
3. \( V_c \) lags I by 90°

According to Krichoff’s laws

**Steps to draw phasor diagram:**
1. Take current I as reference
2. \( V_R \) is in phase with current I
3. \( V_L \) leads current by 90°
4. \( V_c \) lags current by 90°
5. Obtain resultant of \( V_L \) and \( V_c \). Both \( V_L \) and \( V_c \) are in phase opposition (180° out of phase)
6. Add that with \( V_R \) by law of parallelogram to get supply voltage.

The phasor diagram depends on the condition of magnitude of \( V_L \) and \( V_c \) which ultimately depends on values of \( X_L \) and \( X_c \).

Let us consider different cases:

**Case(i):** \( X_L > X_c \)

When \( X_L > X_c \)

Also \( V_L > V_c \) (or) \( IX_L > IX_c \)

So, resultant of \( V_L \) and \( V_c \) will directed towards \( V_L \) i.e. leading current I. Hence I lags V i.e. current I will lags the resultant of \( V_L \) and \( V_c \) i.e. \( V_L - V_c \). The circuit is said to be inductive in nature.

From voltage triangle,

\[
V = \sqrt{V_R^2 + (V_L - V_c)^2} = \sqrt{(IR)^2 + (IX_L - IX_c)^2} \\
V = I \sqrt{(R^2 + (X_L - X_c)^2)} \\
V = IZ \\
Z = \sqrt{(R^2 + (X_L - X_c)^2)} \\
\]

If, \( V = V_m \sin wt \); \( i = I_m \sin (wt - \phi) \)

i.e I lags V by angle \( \phi \)

**Case(ii):** \( X_L < X_c \)

When \( X_L < X_c \)

Also \( V_L < V_c \) (or) \( IX_L < IX_c \)

Hence the resultant of \( V_L \) and \( V_c \) will directed towards \( V_c \) i.e current is said to be capacitive in nature.

Form voltage triangle

\[
V = \sqrt{V_R^2 + (V_c - V_L)^2} = \sqrt{(IR)^2 + (IX_c - IX_L)^2} \\
V = I \sqrt{(R^2 + (X_c - X_L)^2)} \\
V = IZ \\
Z = \sqrt{(R^2 + (X_c - X_L)^2)} \\
\]

If, \( V = V_m \sin wt \); \( i = I_m \sin (wt + \phi) \)

i.e I lags V by angle \( \phi \)

**Case(iii):** \( X_L = X_c \)

When \( X_L = X_c \)

Also \( V_L = V_c \) (or) \( IX_L = IX_c \)

So \( V_L \) and \( V_c \) cancel each other and the resultant is zero. So \( V = V_R \) in such a case, the circuit is purely resistive in nature.

**Impedance:**

In general for RLC series circuit impedance is given by,

\[
Z = R + jX \\
X = X_L - X_c = \text{Total reactance of the circuit} \\
\]

If \( X_L > X_c \); \( X \) is positive & circuit is Inductive

If \( X_L < X_c \); \( X \) is negative & circuit is Capacitive
If $X_L = X_c$ ; $X = 0$ & circuit is purely Resistive

$$\tan \phi = \frac{(X_L - X_c)R}{R/Z}$$

$$\cos \phi = \frac{R}{Z}$$

$$Z = \sqrt{(R^2 + (X_L - X_c)^2)}$$

**Impedance triangle:**

In both cases

$$R = Z \cos \phi$$

$$X = Z \sin \phi$$

**Power and power triangle:**

The average power consumed by circuit is,

$$P_{avg} = (\text{Average power consumed by } R) + (\text{Average power consumed by } L) + (\text{Average power consumed by } C)$$

$$P_{avg} = \text{Power taken by } R = I^2 R$$

$$V = V \cos \phi$$

$$P = VI \cos \phi$$

Thus, for any condition, $X_L > X_c$ or $X_L < X_c$

General power can be expressed as

$$P = \text{Voltage} \times \text{Component in phase with voltage}$$

**Power triangle:**

$S = \text{Apparent power} = I^2 Z = VI$

$P = \text{Real or True power} = VI \cos \phi = \text{Active power}$

$Q = \text{Reactive power} = VI \sin \phi$

4. An alternating current of frequency 60Hz has a maximum value of 12A

1. Write down value of current for instantaneous values
2. Find the value of current after 1/360 second
3. Time taken to reach 9.6A for the first time.

In the above cases assume that time is reckoned as zero when current wave is passing through zero and increase in positive direction.

**Solution:**

Given:

$F = 60Hz$

$Im = 12A$

$W = 2\pi f = 2\pi \times 60 = 377 \text{ rad/sec}$

(i). Equation of instantaneous value is

$$i = Im \sin wt$$

$$i = 12 \sin 377t$$

(ii). $t = 1/360sec$

$$i = 12 \sin (377/360) = 12 \sin 1.0472 = 10.3924 A$$

$$i = 10.3924 A$$

(iii). $i = 9.6 A$

$$9.6 = 12 \sin 377t$$

$$\sin 377t = 0.8$$

$$377t = 0.9272$$

$$t = 2.459 \times 10^{-3} \text{ sec}$$

5. A 50 Hz, alternating voltage of 150V (rms) is applied...
Find the expression for the instantaneous current in each case. Draw the phasor diagram in each case.

**Solution:**

Given,
- \( F = 50 \text{Hz} \)
- \( V = 150 \text{ V} \)
- \( V_m = \sqrt{2} \text{ Vrms} = \sqrt{2} \times 150 = 212.13 \text{ V} \)

**Case (i):**
- \( R = 10 \Omega \)

\[
I_m = \frac{V_m}{R} = \frac{212.13}{10} = 21.213 \text{ A}
\]

For pure resistive current circuit phase different \( \Phi \)

\[
\Phi = 0
\]

\[
i = I_m \sin(\omega t) = I_m \sin(2\pi ft)
\]

\[
i = 21.213 \sin(100 \pi t) \text{ A}
\]

**Phasor diagram:**

**Case (ii):**
- \( L = 0.2 \text{H} \)

\[
X_L = \omega L = 2\pi fL = \pi \times 50 \times 0.2 = 62.83 \Omega
\]

\[
I_m = \frac{V_m X_L}{X_L} = \frac{212.13}{62.83} = 3.37 \text{ A}
\]

\[
\Phi = -90^0 = \frac{\pi}{2} \text{ rad}
\]

In pure Inductive circuit, I lags V by 90 degree

\[
i = I_m \sin((\omega t - \Phi)) \text{ A}
\]

\[
i = 3.37 \sin((100 \pi t - \pi/2)) \text{ A}
\]

**Phasor diagram:**

**Case (iii):**
- \( C = 50 \mu\text{F} \)

\[
X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}
\]

\[
X_C = \frac{1}{(2\pi \times 50 \times 50 \times 10^{-6})} = 63.66 \Omega
\]

\[
I_m = \frac{V_m X_C}{X_C} = \frac{212.13}{63.66} = 3.33 \text{ A}
\]
In pure capacitive circuit, current leads voltage by 90°

\[ \Phi = 90^\circ = \pi/2 \text{ rad} \]

\( i = I_m \sin (\omega t + \Phi) \text{ A} \)

\( i = 3.33 \sin (\omega t + \Phi) \text{ A} \)

\( i = 3.33 \sin (100 \pi t + \pi/2) \text{ A} \)

**Phasor Diagram:**

6. An alternating current \( i = 414 \sin (2\pi \times 50 \times t) \text{ A} \) is passed through a series circuit of a resistance of 100Ω and an inductance of 0.31831 H. Find the expression for the instantaneous values of voltage across, (MAY-08)
   a. The resistance ,
   b. Inductance
   c. Capacitance

**Solution:**

Given

\[ i = 414 \sin (2\pi \times 50 \times t) \text{ A} \]

\( R = 100 \Omega \)

\( L = 0.31831 \text{ H} \)

\( X_L = 2\pi \times 50 \times 0.31831 = 100 \Omega \)

(i) **Voltage across Resistance:**

\[ V_R = iR = 1.414 \sin (2\pi \times 50 \times t) \times 100 \]

\( V_R = 141.4 \sin (2\pi \times 50 \times t) \text{ V} \)

R.m.s value of \( V_R = 141.4/\sqrt{2} = 100 \text{ V} \)

\( \Phi = 0^\circ \)

\( V_R = 100^\circ = 100 + j0 \text{ V} \)

(ii) **Voltage across Inductance:**

\( V_L = i X_L = 1.414 \sin (2\pi \times 50 \times t + 90^\circ) \times 100 \)
\[ V_L = 141.4 \sin (2\pi \times 50 \ t + 90^\circ) \ V \]

R.m.s value of \( V_L = 141.4\sqrt{2} = 100 \ V \), \( \Phi = 90^\circ \)

\[ V_L = 100\angle 90^\circ = 0 - j100 \ V \]

\[ V = V_R + V_L = 100 + j0 + 0 + 100j \]

\[ V = 100 + j100 = 141.42\angle 45^\circ \ V \]

\[ V_m = \sqrt{2} \times 141.42 = 200 \ V \]

\[ V = 200 \sin (2\pi \times 50 \ t + 45^\circ) \ V \]

7. The waveform of the voltage and current of a circuit are given by
\[ e = 120 \sin (314 \ t) \]
\[ i = 10 \sin (314 + \pi/6) \]
Calculate the value of resistance, capacitance which is connected in series to form the circuit. Also, Draw waveforms for current, voltage and phasor diagram. Calculate power consumed by the circuit.

**Solution:**

Given:

\[ V = 120 \sin (314 \ t) ; \quad V_m = 120 \ V ; \quad 2\pi f = 314 ; \quad f = 50 \ Hz \]
\[ i = 10 \sin (314 + \pi/6) ; \quad I_m = 10 \ A ; \quad \Phi = 30^\circ \]

\[ V = V_m\sqrt{2} = 120\sqrt{2} = 84.85 \ V \]
\[ I = I_m\sqrt{2} = 10\sqrt{2} = 7.07 \ A \]
\[ |Z| = V/I = 84.45/7.07 = 12 \ \Omega \]
\[ Z = 12\angle -30^\circ \rightarrow \text{As current leads by} \ 30^\circ \]
Ckt is RC series Circuit is capacitive nature.
\[ \Phi = -\psi \]
\[ Z = R - jXc \]
\[ R = 10.393 ; \quad Xc = 6 \Omega \]
\[ Xc = 1/2\pi fC \]
\[ 6 = 1/(2\pi \times 50 \times C) \]
\[ C = 530.45 \ \text{uF} \]
\[ P = VI \cos \Phi = 84.85 \times 7.07 \times \cos 30^\circ \]
\[ P = 519.52 \]

8. A resistance of 120\ \Omega and a capacitive reactance of 250\ \Omega are connected in series across a AC voltage source. If a current 0.9 A is flowing in the circuit find out,
(i). Power factor
(ii). Supply voltage
(iii). Voltage across resistance and capacitance
(iv). Active power and reactive power (JUN-09)

**Solution:**

Given :

\[ R = 120 \ \Omega \]
9. A series circuit consisting of 25Ω resistor, 64mH inductor and 80uF capacitor to a 110V, 50Hz, Single phase supply as shown in fig. Calculate the current, Voltage across individual element and overall p.f of the circuit. Draw a neat phasor diagram showing \( \vec{I}, \vec{V_R}, \vec{V_L}, \vec{V_c} \) and \( \vec{V} \).
I = V/Z = 3.4580 A  

V_R = \text{IR} \ (3.4580\angle38.20^\circ)(25) = 86.45\angle38.20^\circ \text{ V} 

V_L = I \ j X_L = (3.4580\angle38.20^\circ)(j \ 20.10) 

= (3.4580\angle38.20^\circ)(20.10\angle90^\circ) 

V_L = 69.50\angle128.12^\circ \text{ V} 

V_c = I(\ -j \ X_c) = (3.4580\angle38.20^\circ)(-j \ 39.78) 

= (3.4580\angle38.20^\circ)(38.78\angle-90^\circ) 

= 134.10\angle-15.9^\circ \text{ V} 

V = 110\angle0^\circ \text{ Volts} 

\cos\Phi = \cos38.20^\circ = 0.7858 \text{ Leading} 

10. A series circuit having pure resistance of 40Ω, pure inductance of 50.07mH and a capacitance is connected across a 400V, 50Hz AC supply. This R, L, C combination draws a current of 10A. Calculate (JUN-09)[AU-DEC-10]  

1. Power factor of circuit  
2. Capacitor value 

Solution: 

\[ V = 400 \text{ V} \]  
\[ f = 50 \text{ Hz} \]  
\[ I = 10 \text{ A} \]  
\[ XL = X_L = 2\pi fL = 2\pi \times 50 \times 50.07 \times 10^{-3} = 15.73 \Omega \]  
\[ Xc = 1/(2\pi fC) \]  
\[ Z = \sqrt{R + j(XL - Xc)} \]  
\[ |Z| = \sqrt{(R^2 + (XL - Xc)^2)} \]  
\[ |Z| = |V|/|I| = 400/10 = 40 \Omega \]  
\[ 40 = \sqrt{(40^2 + (15.73 - Xc)^2)} \]  
\[ 40^2 = 40^2 + (15.73 - Xc)^2 \]  
\[ Xc = 15.73 \Omega \]  
\[ 1/2\pi fC = 15.73 \Rightarrow C = 1/(2\pi f \times 15.73) \]  
\[ C = 2.023 \times 10^{-4} \text{ F} \]  
\[ Z = 40 + j(15.73 - 15.73) = 40 + j0 \Omega \]  
\[ Z = 40\angle0^\circ \Omega \]  

Power factor \( \cos\Phi = \cos0^\circ = 1 \). 

11. What is significance of initial conditions? Write a note on initial conditions in basic circuit elements? 

For higher order differential equation, the number of arbitrary constants equals the order of the equation. If these unknowns are to be evaluated for particular solution, other conditions in network must be known. A set of simultaneous equations must be formed containing general solution and some other equations to match number of unknown with equations.
We assume that at reference time \( t=0 \), network condition is changed by switching action. Assume that switch operates in zero time. The network conditions at this instant are called initial conditions in network.

1. **Resistor**:

![Resistor Diagram]

Equation (1) is linear and also time dependent. This indicates that current through resistor changes if applied voltage changes instantaneously. Thus in resistor, change in current is instantaneous as there is no storage of energy in it.

2. **Inductor**:

\[ V_L = L \frac{di_L}{dt} \]

If dc current flows through inductor, \( \frac{di_L}{dt} \) becomes zero as dc current is constant with respect to time. Hence voltage across inductor, \( V_L \) becomes zero. Thus, as far as dc quantities are considered, in steady state, inductor acts as a short circuit.

We can express inductor current in terms of voltage developed across it as

\[ i_L = \frac{1}{L} \int V_L \, dt \]

In above eqn. The limits of integration is from \(-\infty\) to \( \int_{-\infty}^{t} \). Assuming that switching takes place at \( t=0 \), we can split limits into two intervals as \(-\infty\) to \( \int_{-\infty}^{t} \).

\[ i_L = \frac{1}{L} \int_{-\infty}^{0} V_L \, dt + \frac{1}{L} \int_{0}^{t} V_L \, dt \]

\[ i_L = i_L(0^-) + \frac{1}{L} \int_{0}^{t} V_L \, dt \]

at \( t = 0^+ \) we can write \( i_L(0^+) = i_L(0^-) = i_L(0) \)

I through inductor cannot change instantaneously.

3. **Capacitor**
If dc voltage is applied to capacitor, \( \frac{dV_C}{dt} \) becomes zero as dc voltage is constant with respect to time.

Hence the current through capacitor \( i_C \) becomes zero. Thus as far as dc quantities are considered capacitor acts as an open circuit.

\[
V_C = \frac{1}{C} \int i_C dt
\]

\[
V_C = \frac{1}{C} \int_0^t i_C dt
\]

Splitting limits of integration

\[
V_C = \frac{1}{C} \int_{-\infty}^0 i_C dt + \frac{1}{C} \int_0^t i_C dt
\]

At \( t(0^+) \), equation is given by

\[
V_C \uparrow = V_C \downarrow + \frac{1}{C} \int_{0^-}^{0^+} i_C dt
\]

\[
V_C \uparrow = V_C \downarrow
\]

Thus voltage across capacitor can not change instantaneously.

12. What is time constant? Explain time constant in case of series RL circuit.

Or

A series RL circuit with initial current \( I_0 \) in the inductor is connected to a dc voltage \( V \) at \( t = 0 \). Derive the expression for instantaneous current through the Inductor for \( t > 0 \).

Or

Explain in brief about the step response of series RL circuits.

The response or the output of the series RL and RC circuits driven dc excitations is called step response of the network.

Consider that a dc voltage is applied to any general network through a switch \( k \) as shown in fig.
Initially switch k is kept open for very long time. So no voltage is applied to the network. Thus the voltage at input-terminals of network is zero. So we can write voltage across terminals A and B $V(l)$ is zero. When the switch k is closed at $t=0$, the dc voltage $v$ gets applied to the network. The voltage across terminals A and B suddenly or instantaneously rises to voltage $V$. the variation of voltage across terminals +1 and B against time $t$ as shown in fig (b).

In fig (b) it is observed that at $t=0$, there is a step of $V$ volts. Such signal or function is called step function. We can define step function as

\[
\begin{align*}
V & \geq V; t \geq 0 \\
= 0; t < 0
\end{align*}
\]

When the magnitude of the voltage applied is 1 volt then the function is called unit step function.

When the circuits are driven by driving sources, then such circuits are called driven circuits. When the circuits are without such driving sources, then such circuits are called undriven circuits or source free circuits.

**Step response of Driver series RL circuit:-**

Consider a series RL circuit.

At $t=0^-$, switch k is about to close but not fully closed. As voltage is not applied to the circuit, current in the circuit will be zero.

\[
i_k(t^-) = 0
\]

In this current through inductor can not change instantaneously.

\[
i_k(t^+) = 0
\]

Let initial current through inductor can be represented as $I_0$. in above case $I_0$ is zero.
Assume that switch k is closed at $t = 0$.

Apply KVL:

\[
V = IR + L \frac{di}{dt}
\]

\[
\frac{V}{R} = i + \frac{L}{R} \frac{di}{dt}
\]

\[
\frac{V}{R} - i = \frac{L}{R} \frac{di}{dt}
\]
\[ \frac{R}{L} \frac{dt}{di} = \frac{V}{R} - i \]

Integrating both sides,

\[ \int R \frac{dt}{L} = \int \frac{di}{V} \left( \frac{V}{R} - i \right) \]

\[ \Rightarrow \frac{R}{L} t = -\ln \left( \frac{V}{R} - i \right) + k' \]

\( k' \) = integration constant.

To find \( k' \):

At \( t = 0, i = I_0 = 0 \)

\[ \frac{R}{L} \ln \left( \frac{V}{R} \right) = -\ln \left( \frac{V}{R} - 0 \right) + k' \]

\[ \ln \left( \frac{V}{R} \right) = k' \]

Sub values of \( k' \) in eqn 2 we get

\[ \frac{R}{L} t = \ln \left( \frac{V}{R} \right) - \ln \left( \frac{V}{R} - i \right) \]

\[ \frac{R}{L} t = \ln \left( \frac{V}{R} \right) - \ln \left( \frac{V}{R} \right) \cdot \frac{V}{R} - i \]

Take anti log

\[ \frac{V}{e^{\frac{R}{L}} t} = \frac{R}{V} \]

\[ \frac{V}{R} - i = \frac{V}{R} \cdot e^{-\frac{R}{L} t} \]

\[ i = \frac{V}{R} \left[ 1 - e^{-\frac{R}{L} t} \right] \]

Term \( V/R \) = steady state current

\[ -\frac{V}{R} e^{-\frac{R}{L} t} = \]

Transient part of solution of current.
From above fig (a) shows variation of current \( I \) with respect to time \( t \) i.e. current increases exponentially with respect to time. The rising current produces rising flux, which induces emf in coil. According to Lens’s law, the self induced emf opposes the flow of current. Because of this induced emf and its opposition, the current in the coil don’t reach its max value.

The point p shown on graph indicates that current in circuit rises to 0.632 time’s maximum value of current in steady state.

“The time required for the current to rise to the 0.632 of its final value is known as time constant of given RL circuit. The time constant is denoted by \( \tau \)”. Thus for series RL circuit, time constant is

\[
\tau = \frac{R}{L} \text{ sec}
\]

The initial rate of rise of current is large up to first time constant. At later stage, the rate of rise of current reduces.

Theoretically I reach maximum value after infinite time. Voltage across inductor L is given by

\[
V_L = L \frac{di}{dt}
\]

\[
V_L = L \frac{d}{dt} \left[ \frac{V}{R} \left( 1 - e^{-\frac{R}{L}} \right) \right]
\]

\[
V_L = L \left\{ \frac{d}{dt} \left( \frac{V}{R} \right) - \frac{d}{dt} \left( \frac{V}{R} \cdot e^{-\frac{R}{L}} \right) \right\}
\]

\[
V_L = L \left\{ 0 - \left( \frac{V}{R} \right) \left( -\frac{R}{L} e^{-\frac{R}{L}} \right) \right\}
\]

\[
V_L = Ve^{-\frac{R}{L}} \text{ volts}
\]
a. A series RC circuit with initial current $I_0$ in the inductor is connected to a dc voltage $V\,$ at $t = 0$. 

Derive the expression for instantaneous current through the inductor for $t > 0$.

Or

Explain in brief about the step response of series RC circuits.

Consider series RC circuit as shown in fig. The switch $k$ is in open state initially. There is no charge on condenser and $A_0$ voltage across it. At instant $t = 0$, switch $k$ is closed.

Immediately after closing a switch, the capacitor acts as a short circuit, so current at the time of switching is high. A voltage across capacitor is zero at $t = 0^+$. As capacitor acts as a short circuit, the current is maximum and is given by

$$i = \frac{V}{R}\, \text{amp}$$

The current is maximum at $t = 0^+$ which is charging current. As the capacitor starts charging, the voltage across capacitor $V_C$ starts increasing and charging current starts decreasing. After some time, when the capacitor charges to $V$ volts, it achieves steady state. In steady state, it acts as a open circuit so current will be zero finally.

After switching instant applying Kirchoff’s voltage law.

$$V = V_R + V_C$$

Where $V_R$ is voltage across resistor and $V_C$ is voltage across capacitor.

$$\therefore V = iR + V_C$$

But $i$ can be written as

$$i = C \frac{dV_C}{dt}$$

Substituting value of $i$ in equation of voltage $V$

$$V = RC \frac{dV_C}{dt} + V_C$$

This is the first order linear differential equation.

Rearranging the terms in above equation,

$$V - V_C = RC \frac{dV_C}{dt}$$

Separating the variables,

$$\frac{dt}{RC} = \frac{dV_C}{V - V_C}$$

Integrating both sides of above equation, we have

$$\frac{t}{RC} = -\ln \left( V - V_C \right) + k'$$
Where $k'$ is constant of integration

At $t = 0$, there is no voltage across capacitor

$$V_c = 0$$

Substituting in above eqn. We have

$$0 = -\ln \left( \frac{1}{k'} \right) + k'$$

$$\therefore k' = \ln \left( \frac{1}{k'} \right)$$

General solution becomes,

$$\frac{t}{RC} = -\ln \left( \frac{V}{V_c} - \frac{1}{k'} \right) \ln \left( \frac{1}{k'} \right)$$

$$\therefore \frac{t}{RC} = \ln \left[ \frac{V}{V - V_c} \right]$$

$$\therefore \frac{V}{V - V_c} = e^{\frac{t}{RC}}$$

$$V - V_c = V \cdot e^{-\frac{t}{RC}}$$

$$\therefore V_c = V - V \cdot e^{-\frac{t}{RC}}$$

$$\therefore V_c = V - V \cdot e^{-\frac{t}{RC}}$$

Solution for voltage across capacitor

$V$: Steady state value of voltage across capacitor

$e^{\frac{t}{RC}}$: transient portion of voltage across capacitor.

When the steady state is achieved, total charge on the capacitor is $Q$ coulombs.

$$V = \frac{Q}{C}$$

$$V_c = \frac{q}{c}$$

Where $q$ is instant charge

$$\frac{q}{c} = \frac{Q}{C} \left[ 1 - e^{-\frac{t}{RC}} \right]$$

$$q = Q \left[ 1 - e^{-\frac{t}{RC}} \right]$$

Therefore

Thus the charge on the capacitor also behaves similar to voltage across capacitor.

Now current can be expressed as

$$i_R = V - V_c$$

Above equation can be written using KVL

$$i_R = V - \left[ V \left( 1 - e^{-\frac{t}{RC}} \right) \right]$$

$$\therefore i = \frac{V}{R} \cdot e^{-\frac{t}{RC}}$$

So at $t = 0$, $i = \frac{V}{R}$ is maximum current and in steady state it becomes zero.
The variation of voltage across capacitor and charging with respect to time is shown in fig.

The term RC in equation of VC is called time constant and denoted by measured in seconds

\[ t = R \cdot C = \tau \]  \hspace{1cm} \text{Then}

\[ V_C = V_0 e^{-\frac{t}{\tau}} \]

\[ V_C = 0.632 V \]

So time constant of series RC circuit is defined as time required by capacitor voltage to rise from zero to 0.632 of its final steady state value during charging.

Thus, time constant of RC circuit can be defined as time in seconds, during which the voltage across capacitor would reach its final steady state value of its rate of change was maintained constant at its initial value throughout charging period.

1. Explain the analysis of undriven series RL circuits.
Or
Explain the analysis of source free series RL circuits.[AU-JUN-12]

Current decay in source free series RL circuit:

At \( t = 0^+ \), switch k is kept at position ‘a’ for very long time. Thus, the network is in steady state. Initial current through inductor is given as,

\[ i_L(t) = I_0 = \frac{V}{R} = i_L(t) \]

Because current through inductor can not change instantaneously
Assume that at \( t = 0^+ \) switch k is moved to position ‘b’,
Applying KVL,
\[ L \frac{di}{dt} + iR = 0 \]
\[ \therefore L \frac{di}{dt} = -iR \]

Rearranging the terms in above equation by separating variables
\[ \frac{di}{i} = \frac{-R}{L} \, dt \]
Integrating both sides with respect to corresponding variables
\[ \therefore \ln \left( \frac{I}{I_0} \right) = \frac{-R}{L} t \]
Where \( k' \) is constant of integration.

To find \( k' \): -

Form equation 1, at \( t=0, \, i=I_0 \)
Substituting the values in equation 3
\[ \therefore \ln \left( \frac{I}{I_0} \right) = \frac{-R}{L} \ln k' \]
\[ k' = \ln \left( \frac{I}{I_0} \right) \]

Substituting value of \( k' \) from equation 4 in equation 3
\[ \ln \left( \frac{I}{I_0} \right) = \frac{-R}{L} t + \ln \left( \frac{I}{I_0} \right) \]
\[ \ln \left( \frac{I}{I_0} \right) = \frac{-R}{L} t \]
\[ \frac{i}{I_0} = e^{\frac{-R}{L} t} \]
\[ \therefore i = I_0 \cdot e^{\frac{-R}{L} t} \]

fig. shows variation of current \( i \) with respect to time

From the graph, H is clear that current is exponentially decaying. At point P on graph. The current value is (0.363) times its maximum value. The characteristics of decay are determined by values \( R \) and \( L \) which are two parameters of network.

The voltage across inductor is given by
\[ V_L = L \frac{di}{dt} = L \frac{d}{dt} \left[ I_0 \cdot e^{\frac{-R}{L} t} \right] = L \cdot I_0 \left( -\frac{R}{L} \right) \cdot e^{\frac{-R}{L} t} \]
2. Explain about discharging of capacitor through resistor in source free series RC circuit.
   Or
   Explain the analysis of undriven or source free series RC circuits.

Consider network shown in fig. the switch k is moved from position 1 to 2 at reference time t = 0.

Now before switching take place, the capacitor C is fully charged to V volts and it discharges through resistance R. As time passes, charge and hence voltage across capacitor i.e. $V_c$ decreases gradually and hence discharge current also decreases gradually from maximum to zero exponentially.

After switching has taken place, applying kirchoff’s voltage law,

$$0 = V_R + V_C$$

Where $V_R$ is voltage across resistor and $V_C$ is voltage across capacitor.

$$\therefore V_C = -V_R = -i \cdot R$$

$$i = C \frac{dV_C}{dt}$$

$$\therefore V_C = -R \cdot C \frac{dV_C}{dt}$$

Above equation is linear, homogenous first order differential equation. Hence rearranging we have,

$$\frac{dt}{RC} = -\frac{dV_C}{V_C}$$

Integrating both sides of above equation we have

$$\frac{t}{RC} = -\ln V_C + k'$$

Now at $t = 0$, $V_C = V$ which is initial condition, substituting in equation we have,

$$\therefore 0 = -\ln V + k'$$

$$\therefore k' = \ln V$$

Substituting value of $k'$ in general solution, we have

$$\frac{t}{RC} = -\ln V_C + \ln V$$
\[ t = \frac{RC}{V} \ln \frac{V}{V_C} \]

\[ \therefore \frac{V}{V_C} = e^{\frac{t}{RC}} \]

\[ \therefore V_C = V \cdot e^{tRC} \]

\[ V = \frac{Q}{C} \]

Where \( Q \) is total charge on capacitor. Similarly at any instant, \( V_C = q/c \) where \( q \) is instantaneous charge.

\[ q = \frac{Q}{C} e^{\frac{-t}{RC}} \]

So we have,

\[ q = Q \cdot e^{\frac{-t}{RC}} \]

Thus charge behaves similarly to voltage across capacitor. Now discharging current \( i \) is given by

\[ i = \frac{V_C}{R} \]

but \( V_R = V_C \) when there is no source in circuit.

\[ \therefore i = e^{\frac{V}{R}} \]

\[ \therefore i = e^{\frac{t}{RC}} \]

The above expression is nothing but discharge current of capacitor. The variation of this current with respect to time is shown in fig.

This shows that the current is exponentially decaying. At point P on the graph. The current value is \((0.368)\) times its maximum value. The characteristics of decay are determined by values \( R \) and \( C \), which are 2 parameters of network.

For this network, after the instant \( t = 0 \), there is no driving voltage source in circuit, hence it is called undriven RC circuit.
UNIT – V

ANALYSING THREE PHASE CIRCUITS

Three phases balanced/unbalanced voltage sources - Analysis of three phase 3-wire and 4-wire circuits with star and delta connected loads, balanced & unbalanced - Phasor diagram of voltages and currents - Power and power factor measurements in three phase circuits.

5.1 introduction:

a. For household applications, we use single phase AC supply. But industries or big consumers are consuming large amount of power.
b. Single phase supply is not sufficient for producing large amount of power.
c. The large amount of power can be obtained from three phase AC supply.
d. Advantages of Three phase Supply:
   a. Most of the electric power is generated and distributed in three-phase.
   1. The instantaneous power in a three-phase system can be constant.
   2. The amount of power, the three-phase system is more economical that the single-phase.
   b. In fact, the amount of wire required for a three-phase system is less than that required for an equivalent single-phase system
   c. Three phase induction motors are self starting unlike single phase induction motors.
   d. Three phase machines have better power factor and efficiency.
   e. For the same size, the capacity of a three phase machine is higher.

5.2 Phasor diagram of three phase supply:

Star Connection:

Delta Connection:

1. Connection of Three phases supply:

5.3.1 Star connection:

a. The terminals R, Y and B are connected together to form the star point, also called the neutral (N).
b. The lines R, Y and B are connected to the load. If the neutral is connected to the neutral of the load.
c. **Line Voltage:**
   i. The voltage between the any two lines is called line voltage.

1. **Phase Voltage:**
   a. The voltage between the line and neutral point is called phase voltage.

**Relation between the line voltage and phase voltage:**

\[ V_L = \sqrt{3} V_p \]

- **Line current:**
  - The current through the line is called line current.
- **Phase current:**
  - The current through in any phase winding is called Phase current.

**Relation between the line current and phase current:**

\[ I_L = I_{PH} \]

ii. **Delta Connection:**

1) The end of the one winding R is connected to the start of the next phase winding Y, this connection form delta or mesh connection.

**Relation between the line voltage and phase voltage:**

\[ V_L = V_{PH} \]

**Relation between the line current and phase current:**

\[ I_L = I_p \sqrt{3} \]

ii. **Comparison of star and delta connection:**

✓ Loads connected in delta dissipate three times more power than when connected in star to the same supply.

✓ For the same power, the phase currents must be the same for both delta and star connections (since power=\(3I_p^2R_p\)), hence the line current in the delta connected system is greater than the line current in the corresponding star-connected system.
To achieve the same phase current in a star-connected system as in a delta-connected system, the line voltage in the star system is √3 times the line voltage in the delta system.

Thus for a given power transfer, a delta system is associated with larger line currents (and thus larger conductor cross sectional area) and a star system is associated with a larger line voltage (and thus greater insulation).

b. Three phase balanced and unbalanced voltage sources:

i. Balanced Voltage Sources:

5) If the voltage source have the same amplitude and frequency $\omega$ and are out of phase with each other by 120°, the voltage are said to be balanced

ii. Unbalanced Voltage Sources:

6) If the voltage source have the different amplitude and frequency $\omega$ and are out of phase with each other by 120°, the voltage are said to be balanced

iii. Balanced load:

7) A balanced load is one in which the phase impedances are equal in magnitude and in phase

iv. Unbalanced load:

8) An unbalanced load is one in which the phase impedances are different in magnitude and phases.

1. Types of Unbalanced load:

9) Unbalanced 3 wire star connected load

10) Unbalanced 4 wire star connected load

11) Unbalanced 3 wire delta connected load

b. Analysis of 3phase 3 wire with star & delta connected loads:

i. Star Connected load:

Consider a Y-connected load. We will derive the relationships of voltage, current and power for this connection.
Fig. Three phase Y-connection and phasor diagram

Assume that we are given the phase voltages (sequence ABC):

\[ V_{AN} = V_\phi \angle 0^0 ; \quad V_\phi \text{ is the magnitude of phase voltage} \]
\[ V_{BN} = V_\phi \angle -120^0 \]
\[ V_{cN} = V_\phi \angle 120^0 \]

We want to find the line voltages \( V_{AB} \), \( V_{BC} \) and \( V_{CA} \).

Using KVL,

\[ V_{AB} = V_{AN} + V_{NB} \]
\[ = V_{AN} - V_{BN} \]
\[ = V_\phi \angle 0^0 + V_\phi \angle 60^0 \]
\[ = \sqrt{3}V_\phi \angle 30^0 \]

This can be seen in the phasor diagram.

Similarly, you can find the other line voltages as,

\[ V_{BC} = V_{BN} - V_{CN} = V_\phi \angle -120^0 - V_\phi \angle 120^0 = \sqrt{3}V_\phi \angle -90^0 \]
\[ V_{CA} = V_{CN} - V_{AN} = V_\phi \angle 120^0 - V_\phi \angle 0^0 = \sqrt{3}V_\phi \angle 150^0 \]

See the phasor diagram above.

For the Y-connected three phase system, we observe that:

16. Line voltage \( = \sqrt{3} \) Phase Voltage
17. Line current, \( I_L = \) Phase current, \( I_\phi \)
18. Line voltage \( V_{AB} \) is ahead of phase voltage \( V_{AN} \) by \( 30^0 \)
19. Total power, \( P_T = 3 \) Power per phase
\[ = 3(V_\phi I_\phi \cos \theta) \]
\[ = \sqrt{3} V_L I_L \cos \theta \]
20. Similarly, total reactive power \( Q_T = \sqrt{3} V_L I_L \sin \theta \)
21. The apparent power (or VA)\[ |S| = \sqrt{P_T^2 + Q_T^2} = \sqrt{3}V_L I_L \]

Note: The power factor angle \( \theta \) is the angle between phase voltage \( V_{AN} \) and phase current \( I_{AN} \).

5.5.2. Delta connected load:

Consider now a \( \Delta \)-connected load. The circuit connection and phasor diagram showing the voltages and currents for the balanced circuit is shown below.
We assume the phase currents, \( I_{AB} = I_\phi \angle 0^0, I_{BC} = I_\phi \angle -120^0, I_{CA} = I_\phi \angle 120^0 \)

Applying KCL, the line currents are found as,

\[
I_{aa} = I_{AB} - I_{CA} = \sqrt{3}I_\phi \angle -30^0
\]

Similarly,

\[
I_{bb} = \sqrt{3}I_\phi \angle -150^0, I_{cc} = \sqrt{3}I_\phi \angle 90^0
\]

For the \( \Delta \) -connected three phase system, we observe that

13. Line current =\( \sqrt{3} \) Phase current, \( I_\phi \)
14. Line voltage= Phase voltage
15. Line current \( I_{aa} \) is behind phase current \( I_{AB} \) by 30\(^0\)
4) Total power, \( P_T = 3 \) Power per phase
\( = 3 (V_\phi I_\phi \cos \theta) \)
\( = \sqrt{3} V_L I_L \cos \theta \)

13. Similarly, total reactive power \( Q_T = \sqrt{3} V_L I_L \sin \theta \)
14. The apparent power (or VA) = \( |S| = \sqrt{P_T^2 + Q_T^2} = \sqrt{3V_L I_L} \)

Note again: The power factor angle \( \theta \) is the angle between phase voltage \( V_{AB} \) and phase current \( I_{AB} \).

c. Measurements of power & power factor in 3-phase circuits:
i. Measurement of power using single wattmeter:

**Apparent power:**

\[ S = P + jQ = \sqrt{3}V_L I_L \angle \theta \]

**Real power:**

\[ P = 3P_p = 3V_p I_p \cos \theta = \sqrt{3}V_L I_L \cos \theta \]

**Reactive power:**

\[ Q = 3Q_p = 3V_p I_p \sin \theta = \sqrt{3}V_L I_L \sin \theta \]

ii. Two wattmeter method

The connection and phasor diagram are shown for an assumed abc phase sequence and lagging power factor.
Fig. Two-wattmeter method - connection diagram and phasor diagram

The watt-meter readings are given by,

\[ W_1 = V_{ab} I_a \cos \angle (V_{ab}, I_a) = V_L I_L \cos (30^\circ + \theta) \]  

(1)

\[ W_2 = V_{cb} I_c \cos \angle (V_{cb}, I_c) = V_L I_L \cos (30^\circ - \theta) \]  

(2)

The sum of the two watt-meter readings gives the total three phase power,

\[ P_T = W_1 + W_2 = V_L I_L [\cos (30^\circ + \theta) + \cos (30^\circ - \theta)] = \sqrt{3}V_L I_L \]  

(3)

The difference of the two watt-meter readings is

\[ W_2 - W_1 = V_L I_L [\cos (30^\circ - \theta) + \cos (30^\circ + \theta)] = V_L I_L \sin \theta \]  

(4)

The total reactive power is, then,

\[ Q_T = \sqrt{3}(W_2 - W_1) \]  

(5)

The power factor angle can also be found from,
\[ \theta = \tan^{-1} \left( \frac{Q_t}{P_t} \right) = \tan^{-1} \left[ \frac{\sqrt{3} (W_2 - W_1)}{W_2 + W_1} \right] \]  

(6)

Note:

When \( \theta = 0^0 \)  

power factor = 1  

\( W_1 = W_2 \)  

[From eq. (1) and (2)]

\( \theta = 60^0 \)  

pf = 0.5  

\( W_1 = 0, W_2 > 0 \)  

\( ~ \)

\( \theta = 90^0 \)  

pf = 0  

\( W_1 = -W_2 \)  

\( ~ \)

For \( 60^0 \leq \theta \leq 90^0 \), one of the wattmeters will give negative readings. In the laboratory, when you have made the proper wattmeter connections, you will observe that one of the wattmeters is trying to read backwards. After switching the power supply off, reverse the connection of the voltage coil or the current coil (not both). The meter will now read upscale. Assign a negative sign to this reading.

**Advantages of Two wattmeter method:**

1. The method is applicable for balanced as well as unbalanced loads.
2. Only two wattmeter sufficient to measure total 3 phase power.
3. If the load is balanced not only the power but power factor also can be determined.

**Disadvantages:**

1. Not applicable for 3 phases, 4 wire system.
2. The sign of \( W_1 \) & \( W_2 \) must be identified and noted down correctly otherwise it may lead to the wrong result.

**Problems**

1. The input power to a 3-phase a.c. motor is measured as 5kW. If the voltage and current to the motor are 400V and 8.6A respectively, determine the power factor of the system?

Power  

\( P = 5000 \text{W} \),

line voltage \( V_L = 400 \text{V} \),

line current, \( I_L = 8.6 \text{A} \) and

power,  

\( P = \sqrt{3} V_L I_L \cos \phi \)

Hence

**power factor** = \( \cos \phi = \frac{P}{\sqrt{3} V_L I_L} \)

\[ = \frac{5000}{\sqrt{3}} \left( 400 \right) \left( 8.6 \right) \]

\[ = 0.839 \]

2. Two wattmeters are connected to measure the input power to a balanced 3-phase load by the two-wattmeter method. If the instrument readings are 8kW and 4kW, determine (a) the total power input and (b) the load power factor. \( \text{(MAY-08)} \)

(a) Total input power,

\( P = P_1 + P_2 = 8 + 4 = 12 \text{kW} \)

(b) \( \tan \phi = \sqrt{3} (P_1 - P_2) / (P_1 + P_2) \)

\[ = \sqrt{3} \left( \frac{8 - 4}{8 + 4} \right) \]

\[ = \sqrt{3} \left( \frac{4}{12} \right) \]

\[ = \sqrt{3} \left( \frac{1}{3} \right) \]

\[ = \frac{1}{\sqrt{3}} \]

Hence \( \phi = \tan^{-1} \left( \frac{1}{\sqrt{3}} \right) = 30^0 \)

Power factor = \( \cos \phi = \cos 30^0 = 0.866 \)

3. Two wattmeters connected to a 3-phase motor indicate the total power input to be 12kW. The power factor is 0.6. Determine the readings of each wattmeter. \( \text{(MAY-09)} \)

If the two wattmeters indicate \( P_1 \) and \( P_2 \) respectively

Then \( P_1 + P_2 = 12 \text{kW} \)  

\[ ---(1) \]
\[ \tan \phi = \sqrt{3}(P_1 - P_2)/(P_1 + P_2) \]

And power factor = 0.6 = \cos \phi.

Angle \phi = \cos^{-1} 0.6 = 53.13^\circ \quad \text{and} \quad \tan 53.13^\circ = 1.3333.

Hence

\[ 1.3333 = \sqrt{3}(P_1 - P_2)/12 \]

From which,

\[ P_1 - P_2 = 12(1.3333)/\sqrt{3} \]

i.e. \[ P_1 - P_2 = 9.237 \text{kW} \quad \text{----(2)} \]

Adding Equations (1) and (2) gives:

\[ 2P_1 = 21.237 \]

i.e. \[ P_1 = 10.62 \text{kW} \]

Hence wattmeter 1 reads 10.62 kW

From Equation (1), wattmeter 2 reads

\[(12-10.62) = 1.38 \text{kW} \]

4. Three loads, each of resistance 30, are connected in star to a 415 V, 3-phase supply. Determine (a) the system phase voltage, (b) the phase current and (c) the line current. (MAY-09)

A ‘415 V, 3-phase supply’ means that 415 V is the line voltage, VL

(a) For a star connection, VL = \sqrt{3}Vp

Hence phase voltage, \[ Vp = VL/\sqrt{3} = 415/\sqrt{3} = 239.6 \text{ V or 240 V} \]

correct to 3 significant figures

(b) Phase current, Ip = Vp/Rp

\[ = 240/30 = 8 \text{ A} \]

(c) For a star connection, Ip = IL

Hence the line current, IL = 8 A

5. Three identical coils, each of resistance 10\,\Omega and inductance 42mH are connected (a) in star and (b) in delta to a 415V, 50 Hz, 3-phase supply. Determine the total power dissipated in each case.

(a) Star connection

Inductive reactance,

\[ XL = 2\pi f L = 2\pi (50)(42 \times 10^{-3}) = 13.19 \]

Phase impedance,

\[ Zp = \sqrt{R^2 + XL^2} = \sqrt{(10^2 + 13.19^2)} = 16.55 \]

Line voltage,

\[ VL = 415 \text{ V} \]

And phase voltage,

\[ VP = VL/\sqrt{3} = 415/\sqrt{3} = 240 \text{ V} \]

Phase current,

\[ Ip = Vp/Zp = 240/16.55 = 14.50 \text{ A} \]

Line current,

\[ IL = Ip = 14.50 \text{ A} \]

Power factor = \cos \phi = Rp/Zp = 10/16.55 = 0.6042 lagging.

Power dissipated,

\[ P = \sqrt{3} \, VL/IL \cos \phi = \sqrt{3}(415)(14.50)(0.6042) = 6.3 \text{kW} \]

(Alternatively,

\[ P = 3I^2R = 3(14.50)^2(10) = 6.3 \text{kW} \])
(b) Delta connection

$$V_L = V_p = 415 \text{ V},$$

$$Z_p = 16.55, \cos \phi = 0.6042$$

lagging (from above).

Phase current,

$$I_p = \frac{V_p}{Z_p} = \frac{415}{16.55} = 25.08 \text{ A}.$$  

Line current,

$$I_L = \sqrt{3} I_p = \sqrt{3} (25.08) = 43.44 \text{ A}.$$  

Power dissipated,

$$P = \sqrt{3} V_L I_L \cos \phi$$

$$= \sqrt{3} (415)(43.44)(0.6042) = 18.87 \text{ kW}$$

(Alternatively,

$$P = 3 I_p^2 R$$

$$= 3(25.08)^2 (10) = 18.87 \text{ kW}$$)

6. A 415V, 3-phase a.c. motor has a power output of 12.75kW and operates at a power factor of 0.77 lagging and with an efficiency of 85 per cent. If the motor is delta-connected, determine (a) the power input, (b) the line current and (c) the phase current.

(a) Efficiency = power output/power input.

Hence

$$\frac{(85/100)}{12.75} = 10085 \text{ power input from which,}$$

Power input = 12.750 × 10085

$$= 15000 \text{ W or 15 kW}$$

(b) Power, $$P = \sqrt{3} V_L I_L \cos \phi,$$ hence

(c) line current,

$$I_L = \frac{P}{\sqrt{3} (415) (0.77)}$$

$$= \frac{15000}{\sqrt{3} (415) (0.77)}$$

$$= 27.10 \text{ A}$$

(d) For a delta connection, $$I_L = \sqrt{3} I_p,$$

Hence

Phase current, $$I_p = \frac{I_L}{\sqrt{3}}$$

$$= 27.10 / \sqrt{3}$$

$$= 15.65 \text{ A}$$

7. A 400V, 3-phase star connected alternator supplies a delta-connected load, each phase of which has a resistance of 30_ and inductive reactance 40_. Calculate (a) the current supplied by the alternator and (b) the output power and the kVA of the alternator, neglecting losses in the line between the alternator and load.

A circuit diagram of the alternator and load is shown in Fig.

(a) Considering the load:

Phase current, $$I_p = \frac{V_p}{Z_p}$$

Hence $$V_p = 400 \text{ V}.$$  

Phase impedance,

$$Z_p = \sqrt{(R^2 + X^2)}$$

$$= \sqrt{(30^2 + 40^2)} = 50$$
Hence $I_p = \frac{V_p}{Z_p} = \frac{400}{50} = 8\text{A}$.

For a delta-connection,

Line current, $I_L = \sqrt{3} I_p = \sqrt{3} \times 8 = 13.86\text{ A}$.

Hence **13.86A is the current supplied by the alternator**.

(b) Alternator output power is equal to the power Dissipated by the load

I.e. $P = \sqrt{3} V_L I_L \cos \varphi$,

Where $\cos \varphi = \frac{R_p}{Z_p} = \frac{30}{50} = 0.6$.

Hence $P = \sqrt{3} (400) (13.86) (0.6) = 5.76\text{kW}$.

Alternator output kVA,

$S = \sqrt{3} V_L I_L = \sqrt{3} (400) (13.86) = 9.60\text{kVA}$. 
1. **What are the Advantages of 3 phase system?** [DEC-10]
   i. **Most** of the electric power is generated and distributed in three-phase.
   ii. The instantaneous power in a three-phase system can be **constant**.
   iii. The amount of power, the three-phase system is **more economical** that the single-phase.
   iv. In fact, the amount of wire required for a three-phase system is **less than** that required for an equivalent single-phase system.

2. **Define phase, line & neutral?**
   **Phase**
   Describes or pertains to one element or device in a load, line, or source. It is simply a "branch" of the circuit and could look something like this.
   **Line**
   refers to the "transmission line" or wires that connect the source (supply) to the load. It may be modeled as a small impedance (actually 3 of them), or even by just a connecting line.
   **Neutral**
   the 4th wire in the 3-phase system. It's where the phases of a Y connection come together.

3. **Define Phase Voltages & Phase Currents?**
   **Phase Voltages & Phase Currents**
   the voltages and currents across and through a single branch (phase) of the circuit. Note this definition depends on whether the connection is Wye or Delta!

4. **Define line voltage and line current?**
   **Line Currents**
   the currents flowing in each of the lines ($I_a$, $I_b$, and $I_c$). This definition does not change with connection type.

5. **Define line to neutral voltages and line to neutral current?**
   **Line to Neutral Voltages**
   the voltages between any lines and the neutral point ($V_a$, $V_b$, and $V_c$). This definition does not change with connection type, but they may not be physically measurable in a Delta circuit.
   **Line to Neutral Currents**
   same as the line currents ($I_a$, $I_b$, and $I_c$).

6. **Write the relationship of line and phase voltage and current in star?**
   $$ I_L = I_{PH} $$
   $$ V_L = \sqrt{3} V_P $$

7. **Write the relationship of line and phase voltage and current in delta?**
   $$ V_L = V_{PH} $$
   $$ I_L = I_P \sqrt{3} $$

8. **Draw the phasor diagram of delta connection?** (JUN-09)
9. **Define balanced load?**

If the voltage source have the same amplitude and frequency $\omega$ and are out of phase with each other by $120^\circ$, the voltage are said to be balanced.

A **balanced load** is one in which the phase impedances are **equal in magnitude and in phase**

10. **Define unbalanced load? (JUN-09)[DEC-10]**

The load in which the load impedance are not same but having different values. The value of voltage and current are different in each phase.

11. **Types of unbalanced load? (JUN-09)**
   a. Unbalanced 3 wire star connected load
   b. Unbalanced 4 wire star connected load
   c. Unbalanced 3 wire delta connected load.

12. **Write 3 phase power equation?**

Apparent power:

$$S = P + jQ = \sqrt{3}V_LI_L \angle \theta$$

True power:

$$P = 3P_p = 3V_pI_p \cos \theta = \sqrt{3}V_LI_L \cos \theta$$

Reactive power:

$$Q = 3Q_p = 3V_pI_p \sin \theta = \sqrt{3}V_LI_L \sin \theta$$

17. **Write the power factor calculation of two wattmeter method? (JUN-09)**

$$\theta = \tan^{-1}\left(\frac{Q}{P}\right) = \tan^{-1}\left[\sqrt{3}\frac{(W_2 - W_1)}{W_2 + W_1}\right]$$

$$\cos \theta$$,

1. **Draw two wattmeter methods for measurement of power in 3 phase systems?**
2. Comparisons of star and delta connections?
   4. Loads connected in delta dissipate three times more power than when connected in star to the same supply.
   5. For the same power, the phase currents must be the same for both delta and star connections (since power=3I_p^2R_p), hence the line current in the delta connected system is greater than the line current in the corresponding star-connected system.
   6. To achieve the same phase current in a star-connected system as in a delta-connected system, the line voltage in the star system is \( \sqrt{3} \) times the line voltage in the delta system.
   7. Thus for a given power transfer, a delta system is associated with larger line currents (and thus larger conductor cross sectional area) and a star system is associated with a larger line voltage (and thus greater insulation).

3. Three loads, each of resistance 30 \( \Omega \), are connected in star to a 415 V, 3-phase supply. Determine (a) the system phase voltage, (b) the phase current and (c) the line current. (MAY-09)

A ‘415 V, 3-phase supply’ means that 415 V is the line voltage, VL.
(a) For a star connection, VL = \( \sqrt{3} \) Vp
Hence phase voltage, \( V_p = V_L/\sqrt{3} \)
\( = \frac{415}{\sqrt{3}} \)
\( = 239.6 \text{ V or } 240 \text{ V} \)
correct to 3 significant figures
(b) Phase current, \( I_p = V_p/R_p \)
\( = \frac{240}{30} \)
\( = 8 \text{ A} \)
(c) For a star connection, \( I_p = I_L \)
Hence the line current, \( I_L = 8 \text{ A} \)

PART-B
16 MARKS

1. A Three phase delta connected load has \( Z_{ab} = (100+j0) \text{ ohm} \), \( Z_{bc} = (-j100) \text{ ohm} \) and \( Z_{ca} = (70.7+j70.7) \text{ ohm} \) is connected to a balanced 3 phase 400V supply. Determine the line currents \( I_a \), \( I_b \) and \( I_c \). (JAN-09)
2. Prove how 3 phase can be measured using two wattmeters (JAN-09,07)
3. A 3phase balanced delta connected load of 4.3+j7 connected across a 400V 3 phase balanced supply. Determine \( I_{ph} \), \( I_l \) and \( P, Q, S, \text{ p.f.} \). (JAN-09)